Concepts for
Random Number Generation in C++0X

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In anticipation of a National Body comment regarding the recently-approved Committe Draft, this document proposes adjustments to the description of the C++0X Standard Library's random number facility in order to make full use of concepts as formulated in the following documents:

— N2773 = Gregor, *et al.*: **Proposed Wording for Concepts (Revision 9)**,

— N2774 = Gregor, *et al.*: **Foundational Concepts for the C++0X Standard Library (Revision 5)**,

— N2758 = Gregor, *et al.*: **Iterator Concepts for the C++0X Standard Library (Revision 5)**,

— N2776 = Gregor, *et al.*: **Concepts for the C++0X Standard Library: Containers (Revision 4)**,

— N2759 = Gregor, *et al.*: **Concepts for the C++0X Standard Library: Algorithms (Revision 5)**,

We make every attempt to provide complete backward compatibility with the pre-concept Standard Library, and will note each area in which we have knowingly changed semantics.

This document is based on Clause 26.4 [random.numbers] of N2723 = Becker: **Working Draft, Standard for Programming Language C++**. Wherever the numbering of a (sub)section herein matches a (sub)section of that Working Draft, the text in this document should be considered replacement text, except that any relevant issue resolutions or editorial adjustments applied subsequent to that Draft will also need to be applied to this paper, perhaps with some small adjustments in wording.

Wherever possible within the proposed wording herein, adjustments to text will be denoted as added, ~~deleted~~, or ~~changed~~modified; editorial and similar remarks not part of the proposed wording will be shaded. Changes in the index are not specially marked.

We would like to acknowledge the Fermi National Accelerator Laboratory's Computing Division, sponsors of our participation in the C++ standards effort, for its support.

Contents

26 Numerics library [numerics]

26.4 Random number generation [random.numbers]

1 This subclause defines a facility for generating (pseudo-)random numbers.

2 In addition to a few utilities, four categories of entities are described: *uniform random number generators*, *random number engines*, *random number engine adaptors*, and *random number distributions*. These categorizations are applicable to types that satisfy the corresponding requirements, to objects instantiated from such types, and to templates producing such types when instantiated. [*Note:* These entities are specified in such a way as to permit the binding of any uniform random number generator object `e` as the argument to any random number distribution object `d`, thus producing a zero-argument function object such as given by `bind(d,e)`. — *end note*]

3 Each of the entities specified via this subclause has an associated arithmetic type [basic.fundamental] identified as `result_type`. With `T` as the `result_type` thus associated with such an entity, that entity is characterized

a) as *boolean* or equivalently as *boolean-valued*, if `T` is `bool`;

b) otherwise as *integral* or equivalently as *integer-valued*, if `numeric_limits<T>::is_integer` is `true`;

c) otherwise as *floating* or equivalently as *real-valued*.

If integer-valued, an entity may optionally be further characterized as *signed* or *unsigned*, according to `T`.

4 Unless otherwise specified, all descriptions of calculations in this subclause use mathematical real numbers.

5 Throughout this subclause, the operators bitand, bitor, and xor denote the respective conventional bitwise operations. Further,

a) the operator rshift denotes a bitwise right shift with zero-valued bits appearing in the high bits of the result, and

b) the operator lshift_w denotes a bitwise left shift with zero-valued bits appearing in the low bits of the result, and whose result is always taken modulo 2^w.

26.4.1 Requirements [rand.req]

Remove this section 26.4.1 [rand.req] in its entirety; its content has been recast in concept form and for the most part moved into the new section 26.4.3 [rand.concept].

26.4.2 Header `<random>` synopsis [rand.synopsis]

```
namespace std {

  // 26.4.3.1 [rand.concept.urng] Concept UniformRandomNumberGenerator
  concept UniformRandomNumberGenerator<typename U> see below

  // 26.4.3.2 [rand.concept.eng] Concept RandomNumberEngine
  concept RandomNumberEngine<typename E> see below

  // 26.4.3.3 [rand.concept.adapt] Concept RandomNumberEngineAdaptor
  concept RandomNumberEngineAdaptor<typename A> see below

  // 26.4.3.4 [rand.concept.dist] Concept RandomNumberDistribution
  concept RandomNumberDistribution<typename D> see below

  // 26.4.3.5 [rand.concept.seedseq] Concept SeedSequence
  concept SeedSequence<typename S> see below

  // 26.4.4.1 [rand.eng.lcong] Class template linear_congruential_engine
  template<~~class~~UnsignedIntegralLike UIntType, UIntType a, UIntType c, UIntType m>
    requires IntegralType<UIntType>
          && True<m == 0u || (a < m && c < m)>
    class linear_congruential_engine;

  // 26.4.4.2 [rand.eng.mers] Class template mersenne_twister_engine
  template<~~class~~UnsignedIntegralLike UIntType, size_t w, size_t n, size_t m, size_t r,
           UIntType a, size_t u, size_t s,
           UIntType b, size_t t,
           UIntType c, size_t l>
    requires IntegralType<UIntType>
          && True<1u <= m && 1u <= n
               && r <= w && u <= w && s <= w && t <= w && l <= w
               && w <= numeric_limits<UIntType>::digits
               && a <= 2u<<w - 1u && b <= 2u<<w - 1u && c <= 2u<<w - 1u>
    class mersenne_twister_engine;

  // 26.4.4.3 [rand.eng.sub] Class template subtract_with_carry_engine
  template<~~class~~UnsignedIntegralLike UIntType, size_t w, size_t s, size_t r>
    requires IntegralType<UIntType>
          && True<0u < s && s < r && 0 < w && w <= numeric_limits<UIntType>::digits>
    class subtract_with_carry_engine;

  // 26.4.5.1 [rand.adapt.disc] Class template discard_block_engine
  template<~~class~~RandomNumberEngine Engine, size_t p, size_t r>
    requires True<1 <= r && r <= p)>
    class discard_block_engine;

  // 26.4.5.2 [rand.adapt.ibits] Class template independent_bits_engine
  template<~~class~~RandomNumberEngine Engine, size_t w, ~~class~~UnsignedIntegralLike UIntType>
    requires IntegralType<UIntType>
          && True<0u < w && w <= numeric_limits<result_type>::digits>
```

```
  class independent_bits_engine;

// 26.4.5.3 [rand.adapt.shuf] Class template shuffle_order_engine
template<classRandomNumberEngine Engine, size_t k>
  requires True<1u <= k>
  class shuffle_order_engine;

// 26.4.6 [rand.predef] Engines and engine adaptors with predefined parameters
typedef see below minstd_rand0;
typedef see below minstd_rand;
typedef see below mt19937;
typedef see below ranlux24_base;
typedef see below ranlux48_base;
typedef see below ranlux24;
typedef see below ranlux48;
typedef see below knuth_b;
typedef see below default_random_engine;

// 26.4.7 [rand.device] Class random_device
class random_device;

// 26.4.8.1 [rand.util.seedseq] Class seed_seq
class seed_seq;

// 26.4.8.2 [rand.util.canonical] Function template generate_canonical
template<classFloatingPointLike RealType, size_t bits, class UniformRandomNumberGenerator URNG>
  requires FloatingPointType<RealType>
  RealType generate_canonical(UniformRandomNumberGeneratorURNG& g);

// 26.4.9.1.1 [rand.dist.uni.int] Class template uniform_int_distribution
template<classIntegralLike IntType = int>
  requires IntegralType<IntType>
  class uniform_int_distribution;

// 26.4.9.1.2 [rand.dist.uni.real] Class template uniform_real_distribution
template<classFloatingPointLike RealType = double>
  requires FloatingPointType<RealType>
  class uniform_real_distribution;

// 26.4.9.2.1 [rand.dist.bern.bernoulli] Class bernoulli_distribution
class bernoulli_distribution;

// 26.4.9.2.2 [rand.dist.bern.bin] Class template binomial_distribution
template<classIntegralLike IntType = int>
  requires IntegralType<IntType>
  class binomial_distribution;

// 26.4.9.2.3 [rand.dist.bern.geo] Class template geometric_distribution
template<classIntegralLike IntType = int>
  requires IntegralType<IntType>
```

```
  class geometric_distribution;

// 26.4.9.2.4 [rand.dist.bern.negbin] Class template negative_binomial_distribution
template<~~class~~IntegralLike IntType = int>
  requires IntegralType<IntType>
  class negative_binomial_distribution;

// 26.4.9.3.1 [rand.dist.pois.poisson] Class template poisson_distribution
template<~~class~~IntegralLike IntType = int>
  requires IntegralType<IntType>
  class poisson_distribution;

// 26.4.9.3.2 [rand.dist.pois.exp] Class template exponential_distribution
template<~~class~~FloatingPointLike RealType = double>
  requires FloatingPointType<RealType>
  class exponential_distribution;

// 26.4.9.3.3 [rand.dist.pois.gamma] Class template gamma_distribution
template<~~class~~FloatingPointLike RealType = double>
  requires FloatingPointType<RealType>
  class gamma_distribution;

// 26.4.9.3.4 [rand.dist.pois.weibull] Class template weibull_distribution
template<~~class~~FloatingPointLike RealType = double>
  requires FloatingPointType<RealType>
  class weibull_distribution;

// 26.4.9.3.5 [rand.dist.pois.extreme] Class template extreme_value_distribution
template<~~class~~FloatingPointLike RealType = double>
  requires FloatingPointType<RealType>
  class extreme_value_distribution;

// 26.4.9.4.1 [rand.dist.norm.normal] Class template normal_distribution
template<~~class~~FloatingPointLike RealType = double>
  requires FloatingPointType<RealType>
  class normal_distribution;

// 26.4.9.4.2 [rand.dist.norm.lognormal] Class template lognormal_distribution
template<~~class~~FloatingPointLike RealType = double>
  requires FloatingPointType<RealType>
  class lognormal_distribution;

// 26.4.9.4.3 [rand.dist.norm.chisq] Class template chi_squared_distribution
template<~~class~~FloatingPointLike RealType = double>
  requires FloatingPointType<RealType>
  class chi_squared_distribution;

// 26.4.9.4.4 [rand.dist.norm.cauchy] Class template cauchy_distribution
template<~~class~~FloatingPointLike RealType = double>
  requires FloatingPointType<RealType>
```

```
    class cauchy_distribution;

  // 26.4.9.4.5 [rand.dist.norm.f] Class template fisher_f_distribution
  template<classFloatingPointLike RealType = double>
    requires FloatingPointType<RealType>
    class fisher_f_distribution;

  // 26.4.9.4.6 [rand.dist.norm.t] Class template student_t_distribution
  template<classFloatingPointLike RealType = double>
    requires FloatingPointType<RealType>
    class student_t_distribution;

  // 26.4.9.5.1 [rand.dist.samp.discrete] Class template discrete_distribution
  template<classIntegralLike IntType = int>
    requires IntegralType<IntType>
    class discrete_distribution;

  // 26.4.9.5.2 [rand.dist.samp.pconst] Class template piecewise_constant_distribution
  template<classFloatingPointLike RealType = double>
    requires FloatingPointType<RealType>
    class piecewise_constant_distribution;

  // 26.4.9.5.3 [rand.dist.samp.genpdf] Class template general_pdf_distribution
  template<classFloatingPointLike RealType = double>
    requires FloatingPointType<RealType>
    class general_pdf_distribution;
} // namespace std
```

26.4.3 Concepts and related requirements for random number generation [rand.concept]

26.4.3.1 Concept `UniformRandomNumberGenerator` [rand.concept.urng]

1 A *uniform random number generator* `g` of type `G` is a function object returning unsigned integral values such that each value in the range of possible results has (ideally) equal probability of being returned. [*Note:* The degree to which g's results approximate the ideal is often determined statistically. *— end note*]

The following concept may be overconstrained, as its `IntegralType<result_type>` requirement does not allow a URNG that returns a value of user-defined unsigned integral type `T`. We should instead merely require that `T` have the expected unsigned integer behaviors (arithmetic, interoperability, *etc.*). However, additional groundwork seems needed before we can do so. (Papers N2645 = **Fundamental Mathematical Concepts . . .** and N2650 = **Toward a More Complete Taxonomy . . .** explicate some of the necessary concepts.)

```
concept UniformRandomNumberGenerator<typename G> : Callable<G> {
  requires UnsignedIntegralLike<result_type>
        && IntegralType<result_type>;

  static constexpr result_type G::min();
  static constexpr result_type G::max();
```

```
    axiom NonemptyRange(G& g) {
      min() < max();
    }
    axiom InRange(G& g) {
      min() <= g() && g() <= max();
    }
  }
```

```
result_type operator()(G& g);   // from Callable<G>
```

2 *Complexity:* amortized constant.

26.4.3.2 Concept RandomNumberEngine [rand.concept.eng]

1 A *random number engine* (commonly shortened to *engine*) e of type E is a uniform random number generator that additionally meets the requirements (*e.g.*, for seeding and for input/output) specified in this section.

2 Unless otherwise specified, the complexity of each function specified via the RandomNumberEngine concept (including those specified via any less-refined concept) shall be $\mathscr{O}$(size of state).

The input/output requirements of this and subsequent sections are specified in terms of stream concepts InputStreamable and OutputStreamable that are planned for C++0X, but that have not to date been formally proposed.

```
  concept RandomNumberEngine<typename E> : Regular<E>, UniformRandomNumberGenerator<E> {
    requires Constructible<E, result_type>;
    template<SeedSequence Sseq> E::E(Sseq& q);

    void seed(E& e);
    void seed(E& e, result_type s);
    template<SeedSequence Sseq> void seed(E& e, Sseq& q);

    void discard(E& e, unsigned long long z) {
      for( ; z > 0ULL; --z)
        e();
    }

    template<OutputStreamable OS> OS& operator<<(OS& os, const E& e);
    template<InputStreamable IS> IS& operator<<(IS& is, E& e);

    axiom Uniqueness( E& e, E& f, result_type s, Sseq& q) {
      (seed(e)  , e) == (seed(f)  , f);
      (seed(e,s), e) == (seed(f,s), f);
      (seed(e,q), e) == (seed(f,q), f);
    }

    axiom Seeding(E& e, result_type s, Sseq& q) {
      (seed(e)  , e) == E();
      (seed(e,s), e) == E(s);
      (seed(e,q), e) == E(q);
```

```
    }
  }
```

3 At any given time, `e` has a state $\mathtt{e}_i$ for some integer $i \geq 0$. Upon construction, `e` has an initial state $\mathtt{e}_0$. An engine's state may be established via a constructor, a `seed` member function, assignment, or a suitable `operator>>`.

4 `E`'s specification shall define:

a) the size of `E`'s state in multiples of the size of `result_type`, given as an integral constant expression;

b) the *transition algorithm* TA by which `e`'s state $\mathtt{e}_i$ is advanced to its *successor state* $\mathtt{e}_{i+1}$; and

c) the *generation algorithm* GA by which an engine's state is mapped to a value of type `result_type`.

`bool operator==(const E& e1, const E& e2); // from Regular<E>`

5 *Returns:* `true` if $S_1 = S_2$, where S_1 and S_2 are the infinite sequences of values that would be generated, respectively, by repeated future calls to `e1()` and `e2()`. Otherwise returns `false`.

`void operator()(E& e); // from UniformRandomNumberGenerator<E>`

6 *Effects:* Sets the state to $\mathtt{e}_{i+1} = \mathsf{TA}(\mathtt{e}_i)$.

7 *Returns:* $\mathsf{GA}(\mathtt{e}_i)$.

8 *Complexity:* as specified in **??** [rand.req.urng] via the `UniformRandomNumberGenerator` concept.

`E::E(result_type s); // from Constructible<E,result_type>`

9 *Effects:* Creates an engine with an initial state that depends on `s`.

`template<SeedSequence Sseq> E::E(Sseq& q);`

10 *Effects:* Creates an engine with an initial state that depends on a sequence produced by one call to `q.generate`.

11 *Complexity:* Same as complexity of `q.generate` when called on a sequence whose length is size of state.

12 *Note:* This constructor (as well as the corresponding `seed()` function below) may be particularly useful to applications requiring a large number of independent random sequences.

`void seed(E& e);`

13 *Complexity:* same as `E()`.

`void seed(E& e, result_type s);`

14 *Complexity:* same as `E(s)`.

`template<SeedSequence Sseq> void seed(E& e, Sseq& q);`

15 *Complexity:* same as `E(q)`.

`void discard(E& e, unsigned long long z);`

16 *Effects:* Advances the engine's state from $\mathtt{e}_i$ to $\mathtt{e}_{i+\mathtt{z}}$ by any means equivalent to the default implementation specified above.

17 *Complexity:* no worse than the complexity of `z` consecutive calls to `operator()()`.

18 *Note:* This operation is common in user code, and can often be implemented in an engine-specific manner so as to provide significant performance improvements over the default implementation specified above.

```
template<OutputStreamable OS> OS& operator<<(OS& os, const E& e);
```

19 *Effects:* With `os.`*fmtflags* set to `ios_base::dec|ios_base::left` and the fill character set to the space character, writes to `os` the textual representation of `e`'s current state. In the output, adjacent numbers are separated by one or more space characters.

20 *Returns:* the updated `os`.

21 *Postcondition:* The `os.`*fmtflags* and fill character are unchanged.

```
template<InputStreamable IS> IS& operator<<(IS& is, E& e);
```

22 *Requires:* `is` provides a textual representation that was previously written using an output stream whose imbued locale was the same as that of `is`, and whose associated types `OutputStreamable::charT` and `OutputStreamable ::traits` were respectively the same as those of `is`.

23 *Effects:* With `is.fmtflags` set to `ios_base::dec`, sets `e`'s state as determined by reading its textual representation from `is`. If bad input is encountered, ensures that `e`'s state is unchanged by the operation and calls `is.setstate(ios::failbit)` (which may throw `ios::failure` [iostate.flags]). If a textual representation written via `os << x` was subsequently read via `is >> v`, then `x == v` provided that there have been no intervening invocations of `x` or of `v`.

24 *Returns:* the updated `is`.

25 *Postcondition:* The `is.`*fmtflags* are unchanged.

26.4.3.3 Concept `RandomNumberEngineAdaptor` [rand.concept.adapt]

1 A *random number engine adaptor* (commonly shortened to *adaptor*) `a` of type `A` is a random number engine that takes values produced by some other random number engine or engines, and applies an algorithm to those values in order to deliver a sequence of values with different randomness properties. Engines adapted in this way are termed *base engines* in this context. The terms *unary*, *binary*, and so on, may be used to characterize an adaptor depending on the number n of base engines that adaptor utilizes.

2 The base engines of `A` are arranged in an arbitrary but fixed order, and that order is consistently used whenever functions are applied to those base engines in turn. In this context, the notation $\mathtt{b}_i$ denotes the i^{th} of `A`'s base engines, $1 \leq i \leq n$, and $\mathtt{B}_i$ denotes the type of $\mathtt{b}_i$.

```
concept RandomNumberEngineAdaptor<typename A> : RandomNumberEngine<A> {
  requires Constructible<A, const RandomNumberEngine&...>
        && Constructible<A, RandomNumberEngine&&...>;
}
```

```
A::A();   // from RandomNumberEngine<A>
```

3 *Effects:* Each $\mathtt{b}_i$ is initialized, in turn, as if by its respective default constructor.

`bool operator==(const A& a1, const A& a2);` *// from RandomNumberEngine<A>*

4 *Returns:* `true` if each pair of corresponding $\mathtt{b}_i$ are equal. Otherwise returns `false`.

`A::A(result_type s);` *// from RandomNumberEngine<A>*

5 *Effects:* Each $\mathtt{b}_i$ is initialized, in turn, with the next available value from the list $s+0, s+1, \ldots$.

`template<SeedSequence Sseq> void A::A(Sseq& q);` *// from RandomNumberEngine<A>*

6 *Effects:* Each $\mathtt{b}_i$ is initialized, in turn, with `q` as argument.

`void seed(A& a);` *// from RandomNumberEngine<A>*

7 *Effects:* For each $\mathtt{b}_i$, in turn, invokes $\mathtt{b}_i$`.seed()`.

`void seed(A& a, result_type s);` *// from RandomNumberEngine<A>*

8 *Effects:* For each $\mathtt{b}_i$, in turn, invokes $\mathtt{b}_i$`.seed(s)` with the next available value from the list $s+0, s+1, \ldots$.

`template<SeedSequence Sseq> void seed(A& a, Sseq& q);` *// from RandomNumberEngine<A>*

9 *Effects:* For each $\mathtt{b}_i$, in turn, invokes $\mathtt{b}_i$`.seed(q)`.

10 `A` shall also satisfy the following additional requirements:

a) The complexity of each function shall be at most the sum of the complexities of the corresponding functions applied to each base engine.

b) The state of `A` shall include the state of each of its base engines. The size of `A`'s state shall be no less than the sum of the base engines' respective sizes.

c) Copying `A`'s state (*e.g.*, during copy construction or copy assignment) shall include copying, in turn, the state of each base engine of `A`.

d) The textual representation of `A` shall include, in turn, the textual representation of each of its base engines.

e) Any constructor satisfying the requirement `Constructible<A, const RandomNumberEngine&...>` or satisfying the requirement `Constructible<A, RandomNumberEngine&&...>` shall have n or more parameters such that the underlying type of parameter i, $1 \le i \le n$, is $\mathtt{B}_i$, and such that all remaining parameters, if any, have default values. The constructor shall create an engine adaptor initializing each $\mathtt{b}_i$, in turn, with a copy of the value of the corresponding argument.

In addition to the adaptor requirements articulated above, the Working Draft has one more: that each adaptor shall contain typedefs that enumerate the types of each of the adaptor's base engines. The omission from this paper is deliberate.

The omitted requirement was originally intended to make it possible to write generic algorithms over engine adaptors, following the general principle that a template should publish all its template parameters. However, noting that the standard adaptors are all named `*_engine`, it became clear that the salient feature of an adaptor is that it behaves as an engine. That it adapts other engines in order to do so is incidental to an adaptor's generic use.

Other than introspection for its own sake, we have found no use for the omitted typedefs: we have found no algorithms restricted to engines that happen to be adaptors. This paper therefore proposes that the omitted base engine typedefs be permanently deleted from engine adaptor requirements.

26.4.3.4 Concept `RandomNumberDistribution` [rand.concept.dist]

1 A *random number distribution* (commonly shortened to *distribution*) `d` of type `D` is a function object returning values that are distributed according to an associated mathematical *probability density function* $p(z)$ or an associated *discrete probability function* $P(z_i)$. A distribution's specification identifies its associated probability function $p(z)$ or $P(z_i)$.

2 An associated probability function is typically expressed using certain externally-supplied quantities known as the *parameters of the distribution*. Such distribution parameters are identified in this context by writing, for example, $p(z\,|\,a,b)$ or $P(z_i\,|\,a,b)$, to name specific parameters, or by writing, for example, $p(z\,|\,\{\mathtt{p}\})$ or $P(z_i\,|\,\{\mathtt{p}\})$, to denote a distribution's parameters `p` taken as a whole.

Some view this concept's `result_type` as overconstrained because it demands an `ArithmeticType` and thus, for example, disallows returning a container of results. However, the specified behavior is of long standing (see 26.4 [random.numbers] paragraph 3) and so should not be relaxed without additional careful consideration.

```
concept RandomNumberDistribution<typename D> : Regular<D> {
  ArithmeticType result_type;
  Regular param_type;
  requires Constructible<D, const param_type&>;

  void reset(D& d);

  template<UniformRandomNumberGenerator URNG> result_type operator()(D& d, URNG& g);
  template<UniformRandomNumberGenerator URNG> result_type operator()(D& d, URNG& g, const param_type& p);

  param_type param(const D& d);
  void param(D& d, const param_type&);

  result_type min(const D& d);
  result_type max(const D& d);

  template<OutputStreamable OS> OS& operator<<(OS& os, const D& d);
  template<InputStreamable IS> IS& operator>>(IS& is, D& d);
}
```

`Regular param_type;`

3 *Requires:*

a) For each of the constructors of `D` taking arguments corresponding to parameters of the distribution, `param_type` shall provide a corresponding constructor subject to the same requirements and taking arguments identical in number, type, and default values.

b) For each of the member functions of `D` that return values corresponding to parameters of the distribution, `param_type` shall provide a corresponding member function with the identical name, type, and semantics.

c) `param_type` shall provide a declaration of the form `typedef D distribution_type;`.

4 *Remark:* It is unspecified whether `param_type` is declared as a (nested) `class` or via a `typedef`. In this subclause 26.4 [random.numbers], declarations of `D::param_type` are in the form of `typedef`s only for convenience of exposition.

```
D::D(const param_type& p);
```

5 *Effects:* Creates a distribution whose behavior is indistinguishable from that of a distribution newly created directly from the values used to create `p`.

6 *Complexity:* same as `p`'s construction.

```
bool operator==(const D& d1, const D& d2);   // from Regular<D>
```

7 *Returns:* `true` if `d1.param() == d2.param()` and $S_1 = S_2$, where S_1 and S_2 are the infinite sequences of values that would be generated, respectively, by repeated future calls to `d1(g1)` and `d2(g2)` whenever `g1 == g2`. Otherwise returns `false`.

```
void reset(D& d);
```

8 *Effects:* Subsequent uses of the distribution do not depend on values produced by any engine prior to invoking `reset`.

9 *Complexity:* constant.

```
template<UniformRandomNumberGenerator URNG> result_type operator()(D& d, URNG& g);
```

10 *Effects:* With `p` = `param()`, the sequence of numbers returned by successive invocations with the same object `u` is randomly distributed according to the associated probability function $p(z \mid \{\texttt{p}\})$ or $P(z_i \mid \{\texttt{p}\})$.

For distributions `x` and `y` of identical type `D`:

a) The sequence of numbers produced by repeated invocations of `x(u)` shall be independent of any invocation of `os << x` or of any `const` member function of `D` between any of the invocations `x(u)`.

b) If a textual representation is written using `os << x` and that representation is restored into the same or a different object `y` using `is >> y`, repeated invocations of `y(u)` shall produce the same sequence of numbers as would repeated invocations of `x(u)`.

11 *Complexity:* amortized constant number of invocations of `u`.

```
template<UniformRandomNumberGenerator URNG> result_type operator()(D& d, URNG& g, const param_type& p);
```

12 *Effects:* The sequence of numbers returned by successive invocations with the same objects `g` and `p` is randomly distributed according to the associated probability function $p(z \mid \{\texttt{p}\})$ or $P(z_i \mid \{\texttt{p}\})$.

```
param_type param(const D& d);
```

13 *Returns:* a value `p` such that `param(D(p)) == p`.

14 *Complexity:* no worse than the complexity of `D(p)`.

```
void param(D& d, const param_type& p);
```

15 *Postcondition:* `param(D(), p)) == p`.

16 *Complexity:* no worse than the complexity of `D(p)`.

```
result_type min(const D& d);
```

17 *Returns:* the greatest lower bound on the values potentially returned by `operator()`, as determined by the current values of the distribution's parameters.

18 *Complexity:* constant.

```
result_type max(const D& d);
```

19 *Returns:* the least upper bound on the values potentially returned by `operator()`, as determined by the current values of the distribution's parameters.

20 *Complexity:* constant.

```
template<OutputStreamable OS> OS& operator<<(OS& os, const D& d);
```

21 *Effects:* Writes to `os` a textual representation for the parameters and the additional internal data of `d`.

22 *Returns:* the updated `os`.

23 *Postcondition:* The `os`.*fmtflags* and fill character are unchanged.

```
template<InputStreamable IS> IS& operator>>(IS& is, D& d);
```

24 *Requires:* `is` provides a textual representation that was previously written using an output stream whose imbued locale was the same as that of `is`, and whose associated types `OutputStreamable::charT` and `OutputStreamable ::traits` were respectively the same as those of `is`.

25 *Effects:* Restores from `is` the parameters and the additional internal data of `d`. If bad input is encountered, ensures that `d` is unchanged by the operation and calls `is.setstate(ios::failbit)` (which may throw `ios::failure` [iostate.flags]).

26 *Returns:* the updated `is`.

27 *Postcondition:* The `is`.*fmtflags* are unchanged.

26.4.3.5 Concept `SeedSequence` [rand.concept.seedseq]

This section 26.4.3.5 [rand.concept.seedseq] has no precise analog in the Working Draft. Rather, the section abstracts the 26.4.8.1 [rand.util.seedseq] requirements originally imposed on the `seed_seq` class. By creating this separate concept (and adjusting the `RandomNumberEngine` concept accordingly), we give users latitude to provide their own seed sequence types, rather than forcing all users to rely solely on the single `seed_seq` specified in the Working Draft.

Although not rising to the level of a defect, a number of users have voiced concerns regarding the Working Paper's lack of such flexibility. These concerns have to date been addressed (for example, in LWG issue 731) via a gentlemen's agreement that the desired customization point would be provided via a `concept` facility once one became viable. We now provide this subsection to fulfill that promise in a manner consistent with users' long-standing latitude to provide their own interoperable generators, engines, adaptors, and distributions.

1 A *seed sequence* is an object that consumes a sequence of integer-valued data and produces a requested number of unsigned integer values i, $0 \le i < 2^{32}$, based on the consumed data. [*Note:* Such an object provides a mechanism to avoid replication of streams of random variates. This can be useful, for example, in applications requiring large numbers of random number engines. *— end note*]

```
concept SeedSequence<typename S> : Semiregular<S>, DefaultConstructible<S> {
  UnsignedIntegralLike result_type;
  requires IntegralType<result_type>
        && True<sizeof uint32_t <= sizeof result_type>;

  template<InputIterator Iter>
    requires IntegralLike<Iter::value_type>
          && IntegralType<Iter::value_type>
    S::S(Iter begin, Iter end, size_t u = numeric_limits<Iter::value_type>::digits);

  template<RandomAccessIterator Iter>
    requires UnsignedIntegralLike<Iter::value_type>
          && IntegralType<Iter::value_type>
          && True<sizeof uint32_t <= sizeof Iter::value_type>
    void generate(S& q, Iter begin, Iter end);

  size_t size(const S& q);
  template<OutputIterator<auto,const result_type&> Iter>
    void param(const S& q, Iter dest);
}
```

```
template<InputIterator Iter>
  requires IntegralLike<Iter::value_type>
        && IntegralType<Iter::value_type>
  S::S(Iter begin, Iter end,
       size_t u = numeric_limits<Iter::value_type>::digits);
```

2 *Effects:* Constructs a `SeedSequence` object having internal state that depends on some or all of the bits of the supplied sequence `[begin, end)`.

```
template<RandomAccessIterator Iter>
  requires UnsignedIntegralLike<Iter::value_type>
        && IntegralType<Iter::value_type>
        && True<sizeof uint32_t <= sizeof Iter::value_type>
  void generate(S& q, Iter begin, Iter end);
```

3 *Effects:* Does nothing if `begin == end`. Otherwise, fills the supplied range `[begin, end)` with 32-bit quantities that depend on the sequence supplied to the constructor and possibly also on the history of `generate`'s previous

invocations.

```
size_t size(const S& q);
```

4 *Returns:* The number of 32-bit units that would be be returned by a call to `param()`.

```
template<OutputIterator<auto,const result_type&> Iter>
  void param(const S& q, Iter dest);
```

5 *Effects:* Copies to the given destination a sequence of 32-bit units that can be provided to the constructor of a second object of the same type, and that would reproduce in that second object a state indistinguishable from the state of the first object.

26.4.4 Random number engine class templates [rand.eng]

1 Except where specified otherwise, the complexity of all functions specified in the following sections is constant.

2 Except ~~as required by Table clausereftab:RandomEngine~~,where specified otherwise, no function described in this section 26.4.4 [rand.eng] throws an exception.

3 ~~The class templates specified in this section 26.4.4 [rand.eng] satisfy the requirements of random number engine ?? [rand.req.eng]~~For every class `E` instantiated from a template specified in this section 26.4.4 [rand.eng], a concept map `RandomNumberEngine<E>` shall be defined in namespace `std` so as to provide mappings from free functions to the corresponding member functions. Descriptions are provided here only for engine operations ~~on the engines~~ that are not described in 26.4.3.2 [rand.concept.eng] or for operations where there is additional semantic information. Declarations for copy constructors, for copy assignment operators, and for equality and inequality operators are not shown in the synopses.

26.4.4.1 Class template `linear_congruential_engine` [rand.eng.lcong]

1 A `linear_congruential_engine` random number engine produces unsigned integer random numbers. The state x_i of a `linear_congruential_engine` object `x` is of size 1 and consists of a single integer. The transition algorithm is a modular linear function of the form $\mathsf{TA}(x_i) = (a \cdot x_i + c) \bmod m$; the generation algorithm is $\mathsf{GA}(x_i) = x_{i+1}$.

```
template<~~class~~UnsignedIntegralLike UIntType, UIntType a, UIntType c, UIntType m>
  requires IntegralType<UIntType>
        && True<m == 0u || (a < m && c < m)>
  class linear_congruential_engine
{
public:
  // types
  typedef UIntType result_type;

  // engine characteristics
  static const result_type multiplier = a;
  static const result_type increment = c;
  static const result_type modulus = m;
  static constexpr result_type min() { return c == 0u ? 1u: 0u };
  static constexpr result_type max() { return m - 1u };
  static const result_type default_seed = 1u;
```

```
    // constructors and seeding functions
    explicit linear_congruential_engine(result_type s = default_seed);
    template<SeedSequence Sseq> explicit linear_congruential_engine(seed_seqSseq& q);
    void seed(result_type s = default_seed);
    template<SeedSequence Sseq> void seed(seed_seqSseq& q);

    // generating functions
    result_type operator()();
    void discard(unsigned long long z);
};
```

2 ~~UIntType shall denote an unsigned integral type large enough to store values as large as $m-1$.~~ If the template parameter m is 0, the modulus m used throughout this section 26.4.4.1 [rand.eng.lcong] is numeric_limits<result_type>::max() plus 1. [*Note:* ~~The result~~m need not be representable as a value of type result_type. — *end note*] ~~Otherwise, the following relations shall hold: a < m and c < m.~~

3 The textual representation consists of the value of x_i.

```
explicit linear_congruential_engine(result_type s = default_seed);
```

4 *Effects:* Constructs a linear_congruential_engine object. If $c \bmod m$ is 0 and s mod m is 0, sets the engine's state to 1, otherwise sets the engine's state to s mod m.

```
template<SeedSequence Sseq> explicit linear_congruential_engine(seed_seqSseq& q);
```

5 *Effects:* Constructs a linear_congruential_engine object. With $k = \left\lceil \frac{\log_2 m}{32} \right\rceil$ and a an array (or equivalent) of length $k+3$, invokes q.generate($a+0$, $a+k+3$) and then computes $S = \left(\sum_{j=0}^{k-1} a_{j+3} \cdot 2^{32j}\right) \bmod m$. If $c \bmod m$ is 0 and S is 0, sets the engine's state to 1, else sets the engine's state to S.

26.4.4.2 Class template mersenne_twister_engine [rand.eng.mers]

1 A mersenne_twister_engine random number engine[1] produces unsigned integer random numbers in the closed interval $[0, 2^w - 1]$. The state x_i of a mersenne_twister_engine object x is of size n and consists of a sequence X of n values of the type delivered by x; all subscripts applied to X are to be taken modulo n.

2 The transition algorithm employs a twisted generalized feedback shift register defined by shift values n and m, a twist value r, and a conditional xor-mask a. To improve the uniformity of the result, the bits of the raw shift register are additionally *tempered* (*i.e.*, scrambled) according to a bit-scrambling matrix defined by values u, s, b, t, c, and ℓ.

The state transition is performed as follows:

a) Concatenate the upper $w - r$ bits of X_{i-n} with the lower r bits of X_{i+1-n} to obtain an unsigned integer value Y.

b) With $\alpha = a \cdot (Y \,\mathsf{bitand}\, 1)$, set X_i to $X_{i+m-n} \,\mathsf{xor}\, (Y \,\mathsf{rshift}\, 1) \,\mathsf{xor}\, \alpha$.

3 The generation algorithm determines the unsigned integer values z_1, z_2, z_3, z_4 as follows, then delivers z_4 as its result:

a) Let $z_1 = X_i \,\mathsf{xor}\, (X_i \,\mathsf{rshift}\, u)$.

[1] The name of this engine refers, in part, to a property of its period: For properly-selected values of the parameters, the period is closely related to a large Mersenne prime number.

b) Let $z_2 = z_1 \operatorname{xor} \left((z_1 \operatorname{lshift}_w s) \operatorname{bitand} b \right)$.

c) Let $z_3 = z_2 \operatorname{xor} \left((z_2 \operatorname{lshift}_w t) \operatorname{bitand} c \right)$.

d) Let $z_4 = z_3 \operatorname{xor} (z_3 \operatorname{rshift} \ell)$.

```
template<classUnsignedIntegralLike UIntType, size_t w, size_t n, size_t m, size_t r,
           UIntType a, size_t u, size_t s,
           UIntType b, size_t t,
           UIntType c, size_t l>
  requires IntegralType<UIntType>
        && True<1u <= m && 1u <= n
             && r <= w && u <= w && s <= w && t <= w && l <= w
             && w <= numeric_limits<UIntType>::digits
             && a <= 2u<<w - 1u && b <= 2u<<w - 1u && c <= 2u<<w - 1u>
  class mersenne_twister_engine
{
public:
  // types
  typedef UIntType result_type;

  // engine characteristics
  static const size_t word_size = w;
  static const size_t state_size = n;
  static const size_t shift_size = m;
  static const size_t mask_bits = r;
  static const UIntType xor_mask = a;
  static const size_t tempering_u = u;
  static const size_t tempering_s = s;
  static const UIntType tempering_b = b;
  static const size_t tempering_t = t;
  static const UIntType tempering_c = c;
  static const size_t tempering_l = l;
  static constexpr result_type min () { return 0; }
  static constexpr result_type max() { return  2^w - 1; }
  static const result_type default_seed = 5489u;

  // constructors and seeding functions
  explicit mersenne_twister_engine(result_type value = default_seed);
  template<SeedSequence Sseq> explicit mersenne_twister_engine(seed_seqSseq& q);
  void seed(result_type value = default_seed);
  template<SeedSequence Sseq> void seed(seed_seqSseq& q);

  // generating functions
  result_type operator()();
  void discard(unsigned long long z);
};
```

4 ~~The following relations shall hold: $1 \le m \le n$; $0 \le r, u, s, t, l \le w \le$ numeric_limits<result_type>::digits; $0 \le a, b, c \le 2^w - 1$.~~

5 The textual representation of x_i consists of the values of $X_{i-n}, \ldots, X_{i-1}$, in that order.

```
explicit mersenne_twister_engine(result_type value = default_seed);
```

6 *Effects:* Constructs a `mersenne_twister_engine` object. Sets X_{-n} to `value` mod 2^w. Then, iteratively for $i = 1-n, \ldots, -1$, sets X_i to

$$\left[1812433253 \cdot \left(X_{i-1} \,\mathsf{xor}\, \left(X_{i-1} \,\mathsf{rshift}\, (w-2)\right)\right) + i \bmod n\right] \bmod 2^w .$$

7 *Complexity:* $\mathscr{O}(\mathrm{n})$.

```
template<SeedSequence Sseq> explicit mersenne_twister_engine(seed_seqSseq& q);
```

8 *Effects:* Constructs a `mersenne_twister_engine` object. With $k = \lceil w/32 \rceil$ and a an array (or equivalent) of length $n \cdot k$, invokes `q.generate(`$a+0$`, `$a+n\cdot k$`)` and then, iteratively for $i = -n, \ldots, -1$, sets X_i to $\left(\sum_{j=0}^{k-1} a_{k(i+n)+j} \cdot 2^{32j}\right) \bmod 2^w$. Finally, if the most significant $w-r$ bits of X_{-n} are zero, and if each of the other resulting X_i is 0, changes X_{-n} to 2^{w-1}.

26.4.4.3 Class template `subtract_with_carry_engine` [rand.eng.sub]

1 A `subtract_with_carry_engine` random number engine produces unsigned integer random numbers.

2 The state x_i of a `subtract_with_carry_engine` object `x` is of size $\mathscr{O}(\mathrm{r})$, and consists of a sequence X of r integer values $0 \le X_i < m = 2^w$; all subscripts applied to X are to be taken modulo r. The state x_i additionally consists of an integer c (known as the *carry*) whose value is either 0 or 1.

3 The state transition is performed as follows:

a) Let $Y = X_{i-s} - X_{i-r} - c$.

b) Set X_i to $y = Y \bmod m$. Set c to 1 if $Y < 0$, otherwise set c to 0.

[*Note:* This algorithm corresponds to a modular linear function of the form $\mathsf{TA}(\mathrm{x}_i) = (a \cdot \mathrm{x}_i) \bmod b$, where b is of the form $m^r - m^s + 1$ and $a = b - (b-1)/m$. — *end note*]

4 The generation algorithm is given by $\mathsf{GA}(\mathrm{x}_i) = y$, where y is the value produced as a result of advancing the engine's state as described above.

```
template<classUnsignedIntegralLike UIntType, size_t w, size_t s, size_t r>
  requires IntegralType<UIntType>
        && True<0u < s && s < r && 0 < w && w <= numeric_limits<UIntType>::digits>
  class subtract_with_carry_engine
{
public:
  // types
  typedef UIntType result_type;

  // engine characteristics
  static const size_t word_size = w;
  static const size_t short_lag = s;
  static const size_t long_lag = r;
  static constexpr result_type min() { return 0; }
  static constexpr result_type max() { return m-1; }
```

```
    static const result_type default_seed = 19780503u;

    // constructors and seeding functions
    explicit subtract_with_carry_engine(result_type value = default_seed);
    template<SeedSequence Sseq> explicit subtract_with_carry_engine(~~seed_seq~~Sseq& q);
    void seed(result_type value = default_seed);
    template<SeedSequence Sseq> void seed(~~seed_seq~~Sseq& q);

    // generating functions
    result_type operator()();
    void discard(unsigned long long z);
};
```

5 ~~The following relations shall hold: $0 < s < r$, and $0 < w \leq$ `numeric_limits<result_type>::digits`.~~

6 The textual representation consists of the values of $X_{i-r}, \ldots, X_{i-1}$, in that order, followed by c.

```
explicit subtract_with_carry_engine(result_type value = default_seed);
```

7 *Effects:* Constructs a `subtract_with_carry_engine` object. Sets the values of $X_{-r}, \ldots, X_{-1}$, in that order, as ~~required~~specified below. If X_{-1} is then 0, sets c to 1; otherwise sets c to 0.

To set the X_k, ~~F~~first construct `e`, a `linear_congruential_engine` object, as if by the following definition:

```
linear_congruential_engine<result_type,
                           40014u,0u,2147483563u> e(value == 0u ? default_seed : value);
```

Then, to set an X_k, use new values $z_0, \ldots, z_{n-1}$ obtained from n successive invocations of `e` taken modulo 2^{32}. Set X_k to $\left(\sum_{j=0}^{n-1} z_j \cdot 2^{32j}\right) \bmod m$. If X_{-1} is then 0, sets c to 1; otherwise sets c to 0.

8 *Complexity:* Exactly $n \cdot$ `r` invocations of `e`.

```
template<SeedSequence Sseq> explicit subtract_with_carry_engine(~~seed_seq~~Sseq& q);
```

9 *Effects:* Constructs a `subtract_with_carry_engine` object. With $k = \lceil w/32 \rceil$ and a an array (or equivalent) of length $r \cdot k$, invokes `q.generate`$(a+0,\ a+r\cdot k)$ and then, iteratively for $i = -r, \ldots, -1$, sets X_i to $\left(\sum_{j=0}^{k-1} a_{k(i+r)+j} \cdot 2^{32j}\right) \bmod m$. If X_{-1} is then 0, sets c to 1; otherwise sets c to 0.

26.4.5 Random number engine adaptor class templates [rand.adapt]

1 Except where specified otherwise, the complexity of all functions specified in the following sections is constant.

2 Except ~~as required by Table ??,~~where specified otherwise, no function described in this section 26.4.5 [rand.adapt] throws an exception.

3 ~~The class templates specified in this section 26.4.5 satisfy the requirements of random number engine adaptor [rand.req.adapt]~~For every class `A` instantiated from a template specified in this section 26.4.5 [rand.adapt], a concept map `RandomNumberEngineAdaptor<E>` shall be defined in namespace `std` so as to provide mappings from free functions to the corresponding member functions. Descriptions are provided here only for adaptor operations ~~on the engines~~ that are not described in section 26.4.3.3 [rand.concept.adapt] or for operations where there is additional semantic information.

Declarations for copy constructors, for copy assignment operators, and for equality and inequality operators are not shown in the synopses.

26.4.5.1 Class template `discard_block_engine` [rand.adapt.disc]

1 A `discard_block_engine` random number engine adaptor produces random numbers selected from those produced by some base engine e. The state $\mathtt{x}_i$ of a `discard_block_engine` engine adaptor object `x` consists of the state $\mathtt{e}_i$ of its base engine `e` and an additional integer n. The size of the state is the size of e's state plus 1.

2 The transition algorithm discards all but $r > 0$ values from each block of $p \geq r$ values delivered by e. The state transition is performed as follows: If $n \geq r$, advance the state of `e` from $\mathtt{e}_i$ to $\mathtt{e}_{i+p-r}$ and set n to 0. In any case, then increment n and advance `e`'s then-current state $\mathtt{e}_j$ to $\mathtt{e}_{j+1}$.

3 The generation algorithm yields the value returned by the last invocation of `e()` while advancing `e`'s state as described above.

```
template<~~class~~RandomNumberEngine Engine, size_t p, size_t r>
  requires True<1 <= r && r <= p)>
  class discard_block_engine
{
public:
  // types
  ~~typedef Engine base_type;~~
  typedef typename ~~base_type~~Engine::result_type result_type;

  // engine characteristics
  static const size_t block_size = p;
  static const size_t used_block = r;
  static constexpr result_type min() { return ~~base_type~~Engine::min; }
  static constexpr result_type max() { return ~~base_type~~Engine::max; }

  // constructors and seeding functions
  discard_block_engine();
  explicit discard_block_engine(const ~~base_type~~Engine& e);
  explicit discard_block_engine(Engine&& e);
  explicit discard_block_engine(result_type s);
  template<SeedSequence Sseq> explicit discard_block_engine(~~seed_seq~~Sseq& q);
  void seed();
  void seed(result_type s);
  template<SeedSequence Sseq> void seed(~~seed_seq~~Sseq& q);

  // generating functions
  result_type operator()();
  void discard(unsigned long long z);

  // property functions
  const ~~base_type~~Engine& base() const;

private:
  ~~base_type~~Engine e;    // exposition only
  int n;             // exposition only
```

```
};
```

4 ~~The following relations shall hold: $1 \le$ r $\le$ p.~~

5 The textual representation consists of the textual representation of `e` followed by the value of `n`.

6 In addition to its behavior pursuant to section 26.4.3.3 [rand.concept.adapt], each constructor that is not a copy constructor sets `n` to 0.

26.4.5.2 Class template `independent_bits_engine` [rand.adapt.ibits]

1 An `independent_bits_engine` random number engine adaptor combines random numbers that are produced by some base engine e, so as to produce random numbers with a specified number of bits w. The state $\mathtt{x}_i$ of an `independent_bits_engine` engine adaptor object `x` consists of the state $\mathtt{e}_i$ of its base engine `e`; the size of the state is the size of e's state.

2 The transition and generation algorithms are described in terms of the following integral constants:

a) Let $R =$ `e.max() - e.min() + 1` and $m = \lfloor \log_2 R \rfloor$.

b) With n as determined below, let $w_0 = \lfloor w/n \rfloor$, $n_0 = n - w \bmod n$, $y_0 = 2^{w_0} \lfloor R/2^{w_0} \rfloor$, and $y_1 = 2^{w_0+1} \lfloor R/2^{w_0+1} \rfloor$.

c) Let $n = \lceil w/m \rceil$ if and only if the relation $R - y_0 \le \lfloor y_0/n \rfloor$ holds as a result. Otherwise let $n = 1 + \lceil w/m \rceil$.

[*Note:* The relation $w = n_0 w_0 + (n - n_0)(w_0 + 1)$ always holds. — *end note*]

3 The transition algorithm is carried out by invoking `e()` as often as needed to obtain n_0 values less than y_0 +`e.min()` and $n - n_0$ values less than $y_1 +$ `e.min()`.

4 The generation algorithm uses the values produced while advancing the state as described above to yield a quantity S obtained as if by the following algorithm:

```
S = 0;
for (k = 0; k ≠ n₀; k += 1)  {
  do u = e() - e.min(); while (u ≥ y₀);
  S = 2^w₀ · S + u mod 2^w₀;
}
for (k = n₀; k ≠ n; k += 1)  {
  do u = e() - e.min(); while (u ≥ y₁);
  S = 2^(w₀+1) · S + u mod 2^(w₀+1);
}
```

```
template<~~class~~RandomNumberEngine Engine, size_t w, ~~class~~UnsignedIntegralLike UIntType>
  requires IntegralType<UIntType>
        && True<0u < w && w <= numeric_limits<result_type>::digits>
  class independent_bits_engine
{
public:
  // types
  ~~typedef Engine base_type;~~
  typedef UIntType result_type;

  // engine characteristics
```

```
    static constexpr result_type min() { return 0; }
    static constexpr result_type max() { return 2^w − 1; }

    // constructors and seeding functions
    independent_bits_engine();
    explicit independent_bits_engine(const ~~base_type~~Engine& e);
    explicit independent_bits_engine(Engine&& e);
    explicit independent_bits_engine(result_type s);
    template<SeedSequence Sseq> explicit independent_bits_engine(~~seed_seq~~Sseq& q);
    void seed();
    void seed(result_type s);
    template<SeedSequence Sseq> void seed(~~seed_seq~~Sseq& q);

    // generating functions
    result_type operator()();
    void discard(unsigned long long z);

    // property functions
    const ~~base_type~~Engine& base() const;

  private:
    ~~base_type~~Engine e;    // exposition only
  };
```

5 ~~The following relations shall hold: $0 < w \le$ `numeric_limits<result_type>::digits`.~~

6 The textual representation consists of the textual representation of `e`.

26.4.5.3 Class template `shuffle_order_engine` [rand.adapt.shuf]

1 A `shuffle_order_engine` random number engine adaptor produces the same random numbers that are produced by some base engine e, but delivers them in a different sequence. The state $\mathtt{x}_i$ of a `shuffle_order_engine` engine adaptor object `x` consists of the state $\mathtt{e}_i$ of its base engine `e`, an additional value Y of the type delivered by `e`, and an additional sequence V of k values also of the type delivered by `e`. The size of the state is the size of e's state plus $k+1$.

2 The transition algorithm permutes the values produced by e. The state transition is performed as follows:

a) Calculate an integer j as $\left\lfloor \frac{k\cdot(Y-b_{\min})}{b_{\max}-b_{\min}+1} \right\rfloor$.

b) Set Y to V_j and then set V_j to `b()`.

3 The generation algorithm yields the last value of `Y` produced while advancing `e`'s state as described above.

```
  template<~~class~~RandomNumberEngine Engine, size_t k>
    requires True<1u <= k>
    class shuffle_order_engine
  {
  public:
    // types
    ~~typedef Engine base_type;~~
```

```
    typedef typename ~~base_type~~Engine::result_type result_type;

    // engine characteristics
    static const size_t table_size = k;
    static constexpr result_type min() { return ~~base_type~~Engine::min; }
    static constexpr result_type max() { return ~~base_type~~Engine::max; }

    // constructors and seeding functions
    shuffle_order_engine();
    explicit shuffle_order_engine(const ~~base_type~~Engine& e);
    explicit shuffle_order_engine(Engine&& e);
    explicit shuffle_order_engine(result_type s);
    template<SeedSequence Sseq> explicit shuffle_order_engine(~~seed_seq~~Sseq& q);
    void seed();
    void seed(result_type s);
    template<SeedSequence Sseq> void seed(~~seed_seq~~Sseq& q);

    // generating functions
    result_type operator()();
    void discard(unsigned long long z);

    // property functions
    const ~~base_type~~Engine& base() const;

  private:
    ~~base_type~~Engine e;    // exposition only
    result_type Y;        // exposition only
    result_type V[k];     // exposition only
  };
```

4 ~~The following relation shall hold: $1 \leq$ k.~~

5 The textual representation consists of the textual representation of `e`, followed by the `k` values of V, followed by the value of Y.

6 In addition to its behavior pursuant to section 26.4.3.3 [rand.concept.adapt], each constructor that is not a copy constructor initializes `V[0], ..., V[k-1]` and Y, in that order, with values returned by successive invocations of `e()`.

26.4.6 Engines and engine adaptors with predefined parameters [rand.predef]

```
typedef linear_congruential_engine<uint_fast32_t, 16807, 0, 2147483647>
        minstd_rand0;
```

1 *Required behavior:* The 10000$^{\text{th}}$ consecutive invocation of a default-constructed object of type `minstd_rand0` shall produce the value 1043618065.

```
typedef linear_congruential_engine<uint_fast32_t, 48271, 0, 2147483647>
        minstd_rand;
```

2 *Required behavior:* The 10000$^{\text{th}}$ consecutive invocation of a default-constructed object of type `minstd_rand` shall produce the value 399268537.

```
typedef mersenne_twister_engine<uint_fast32_t,
                                32,624,397,31,0x9908b0df,11,7,0x9d2c5680,15,0xefc60000,18>
        mt19937;
```

3 *Required behavior:* The 10000$^{\text{th}}$ consecutive invocation of a default-constructed object of type `mt19937` shall produce the value 4123659995.

```
typedef subtract_with_carry_engine<uint_fast32_t, 24, 10, 24>
        ranlux24_base;
```

4 *Required behavior:* The 10000$^{\text{th}}$ consecutive invocation of a default-constructed object of type `ranlux24_base` shall produce the value 7937952.

```
typedef subtract_with_carry_engine<uint_fast64_t, 48, 5, 12>
        ranlux48_base;
```

5 *Required behavior:* The 10000$^{\text{th}}$ consecutive invocation of a default-constructed object of type `ranlux48_base` shall produce the value 61839128582725.

```
typedef discard_block_engine<ranlux24_base, 223, 23>
        ranlux24;
```

6 *Required behavior:* The 10000$^{\text{th}}$ consecutive invocation of a default-constructed object of type `ranlux24` shall produce the value 9901578.

```
typedef discard_block_engine<ranlux48_base, 389, 11>
        ranlux48
```

7 *Required behavior:* The 10000$^{\text{th}}$ consecutive invocation of a default-constructed object of type `ranlux48` shall produce the value 249142670248501.

```
typedef shuffle_order_engine<minstd_rand0,256>
        knuth_b;
```

8 *Required behavior:* The 10000$^{\text{th}}$ consecutive invocation of a default-constructed object of type `knuth_b` shall produce the value 1112339016.

```
typedef implementation-defined
        default_random_engine;
```

9 *Required behavior:* ~~The named entity shall meet the requirements of a Random Number Engine ([rand.concept.eng])~~A concept map `RandomNumberEngine<default_random_engine>`, or equivalent, shall be defined in namespace `std` so as to provide mappings from free functions to the corresponding member functions.

> The above isn't really *required behavior*, but it's not clear how better to label the paragraph.

10 *Remark:* The choice of engine type named by this `typedef` is implementation defined. [*Note:* The implementation may select this type on the basis of performance, size, quality, or any combination of such factors, so as to provide at least acceptable engine behavior for relatively casual, inexpert, and/or lightweight use. Because different

implementations may select different underlying engine types, code that uses this `typedef` need not generate identical sequences across implementations. — *end note*]

26.4.7 Class `random_device` [rand.device]

1 A `random_device` uniform random number generator produces non-deterministic random numbers. ~~It satisfies the requirements of uniform random number generator [rand.req.urng]~~a concept map `UniformRandomNumberGenerator<random_device>` shall be defined in namespace `std` so as to provide mappings from free functions to the corresponding member functions.

2 If implementation limitations prevent generating non-deterministic random numbers, the implementation may employ a random number engine.

```
class random_device
{
public:
  // types
  typedef unsigned int result_type;

  // generator characteristics
  static constexpr result_type min() { return ~~see below~~numeric_limits<result_type>::min(); }
  static constexpr result_type max() { return ~~see below~~numeric_limits<result_type>::max(); }

  // constructors
  explicit random_device(const string& token = implementation-defined);

  // generating functions
  result_type operator()();

  // property functions
  double entropy() const;

  // no copy functions
  random_device(const random_device& ) = delete;
  void operator=(const random_device& ) = delete;
};
```

3 ~~The values of the `min` and `max` members are identical to the values returned by `numeric_limits<result_type>::min)` and `numeric_limits<result_type>::max)`, respectively.~~

```
explicit random_device(const string& token = implementation-defined);
```

4 *Effects:* Constructs a `random_device` non-deterministic uniform random number generator object. The semantics and default value of the `token` parameter are implementation-defined.[2]

5 *Throws:* A value of an implementation-defined type derived from `exception` if the `random_device` could not be initialized.

```
double entropy() const;
```

[2] The parameter is intended to allow an implementation to differentiate between different sources of randomness.

6 *Returns:* If the implementation employs a random number engine, returns 0.0. Otherwise, returns an entropy estimate[3)] for the random numbers returned by `operator()`, in the range `min()` to $\log_2(\texttt{max()}+1)$.

7 *Throws:* Nothing.

```
result_type operator()();
```

8 *Returns:* A non-deterministic random value, uniformly distributed between `min()` and `max()`, inclusive. It is implementation-defined how these values are generated.

9 *Throws:* A value of an implementation-defined type derived from `exception` if a random number could not be obtained.

26.4.8 Utilities [rand.util]

26.4.8.1 Class `seed_seq` [rand.util.seedseq]

1 ~~An object of type seed_seq consumes a sequence of integer-valued data and produces a fixed number of unsigned integer values, $0 \le i < 2^{32}$, based on the consumed data.~~ [*Note:* ~~Such an object provides a mechanism to avoid replication of streams of random variates. This can be useful in applications requiring large numbers of random number engines.~~ — *end note*]

2 ~~In addition to the requirements set forth below, instances of seed_seq shall meet the requirements of CopyConstructible [copyconstructible] and of Assignable [container.requirements].~~

3 No function described in this section 26.4.8.1 [rand.util.seedseq] throws an exception.

4 A concept map `SeedSequence<seed_seq>` shall be defined in namespace `std` so as to provide mappings from free functions to the corresponding member functions.

```
class seed_seq
{
public:
  // types
  typedef uint_least32_t result_type;

  // constructors
  seed_seq();
  template<class InputIterator Iter>
    requires IntegralLike<Iter::value_type>
          && IntegralType<Iter::value_type>
    seed_seq(InputIteratorIter begin, InputIteratorIter end,
             size_t u = numeric_limits<typename iterator_traits<InputIterator>Iter::value_type>::digits);

  // generating functions
  template<class RandomAccessIterator Iter>
    requires UnsignedIntegralLike<Iter::value_type>
          && IntegralType<Iter::value_type>
          && True<sizeof uint32_t <= sizeof Iter::value_type>
    void generate(RandomAccessIteratorIter begin, RandomAccessIteratorIter end) const;
```

3) If a device has n states whose respective probabilities are $P_0, \ldots, P_{n-1}$, the device entropy S is defined as $S = -\sum_{i=0}^{n-1} P_i \cdot \log P_i$.

```
    // property functions
    size_t size() const;
    template<~~class~~ OutputIterator<auto,const result_type&> Iter>
      void param(~~OutputIterator~~Iter dest) const;

  private:
    vector<result_type> v;    // exposition only
  };
```

```
explicit seed_seq();
```

5 *Effects:* Constructs a `seed_seq` object as if by default-constructing its member `v`.

6 ~~*Throws:* Nothing.~~

```
template<~~class~~ InputIterator Iter>
  requires IntegralLike<Iter::value_type>
        && IntegralType<Iter::value_type>
  seed_seq(~~InputIterator~~Iter begin, ~~InputIterator~~Iter end,
           size_t u = numeric_limits<~~typename iterator_traits<InputIterator>~~Iter::value_type>::digits);
```

7 ~~*Requires:* `InputIterator` shall satisfy the requirements of an input iterator [input.iterator] such that `iterator_traits<InputIterator>::value_tye` shall denote an integral type.~~

8 *Effects:* Constructs a `seed_seq` object by rearranging some or all of the bits of the supplied sequence `[begin, end)` of w-bit quantities into 32-bit units, as if by the following:

First extract the rightmost `u` bits from each of the $n =$ `end` $-$ `begin` elements of the supplied sequence and concatenate all the extracted bits to initialize a single (possibly very large) unsigned binary number, $b = \sum_{i=0}^{n-1}(\texttt{begin}[i] \bmod 2^u) \cdot 2^{w \cdot i}$ (in which the bits of each `begin[`i`]` are treated as denoting an unsigned quantity). Then carry out the following algorithm:

```
  v.clear();
  if (w < 32)
    v.push_back(n);
  for( ; n > 0; --n )
    v.push_back(b mod 2^32), b /= 2^32;
```

```
template<~~class~~ RandomAccessIterator Iter>
  requires UnsignedIntegralLike<Iter::value_type>
        && IntegralType<Iter::value_type>
        && True<sizeof uint32_t <= sizeof Iter::value_type>
  void generate(~~RandomAccessIterator~~Iter begin, ~~RandomAccessIterator~~Iter end) ~~const~~;
```

9 ~~*Requires:* `RandomAccessIterator` shall meet the requirements of a random access iterator [random.access.iterators] such that `iterator_traits<RandomAccessIterator>::value_tye` shall denote an unsigned integral type capable of accommodating 32-bit quantities.~~

10 *Effects:* Does nothing if `begin == end`. Otherwise, with $s =$ `v.size()` and $n =$ `end` $-$ `begin`, fills the supplied range $[$`begin`, `end`$)$ according to the following algorithm in which each operation is to be carried out modulo 2^{32}, each indexing operator applied to `begin` is to be taken modulo n, and $T(x)$ is defined as $x \operatorname{xor} (x \operatorname{rshift} 27)$:

a) By way of initialization, set each element of the range to the value `0x8b8b8b8b`. Additionally, for use in subsequent steps, let $p = (n-t)/2$ and let $q = p+t$, where

$$t = (n \geq 623) \ ?\ 11 : (n \geq 68) \ ?\ 7 : (n \geq 39) \ ?\ 5 : (n \geq 7) \ ?\ 3 : (n-1)/2;$$

b) With m as the larger of $s+1$ and n, transform the elements of the range: iteratively for $k = 0, \ldots, m-1$, calculate values

$$\begin{aligned} r_1 &= 1664525 \cdot \mathtt{T}(\mathtt{begin}[k] \operatorname{xor} \mathtt{begin}[k+p] \operatorname{xor} \mathtt{begin}[k-1]) \\ r_2 &= r_1 + \begin{cases} s & , k = 0 \\ k \bmod n + \mathtt{v}[k-1] & , 0 < k \leq s \\ k \bmod n & , s < k \end{cases} \end{aligned}$$

and, in order, increment `begin`$[k+p]$ by r_1, increment `begin`$[x+q]$ by r_2, and set `begin[k]` to r_2.

c) Transform the elements of the range three more times, beginning where the previous step ended: iteratively for $k = m, \ldots, m+n-1$, calculate values

$$\begin{aligned} r_3 &= 1566083941 \cdot \mathtt{T}(\mathtt{begin}[k] + \mathtt{begin}[k+p] + \mathtt{begin}[k-1]) \\ r_4 &= r_3 - (k \bmod n) \end{aligned}$$

and, in order, update `begin`$[k+p]$ by xoring it with r_4, update `begin`$[k+q]$ by xoring it with r_3, and set `begin`$[k]$ to r_4.

11 ~~*Throws:* Nothing.~~

```
size_t size() const;
```

12 *Returns:* The number of 32-bit units that would be be returned by a call to `param()`.

```
template<class OutputIterator<auto,const result_type&> Iter>
  void param(OutputIteratorIter dest) const;
```

13 ~~*Requires:* `OutputIterator` shall satisfy the requirements of an output iterator [output.iterator] such that `iterator_traits<OutputIterator>::value_tye` shall be assignable from `result_type`.~~

14 *Effects:* Copies the sequence of prepared 32-bit units to the given destination, as if by executing the following statement:

```
copy(v.begin(), v.end(), dest);
```

26.4.8.2 Function template `generate_canonical` [rand.util.canonical]

1 Each function instantiated from the template described in this section 26.4.8.2 [rand.util.canonical] maps the result of one or more invocations of a supplied uniform random number generator `g` to one member of the specified `RealType` such that, if the values g_i produced by `g` are uniformly distributed, the instantiation's results t_j, $0 \leq t_j < 1$, are distributed as uniformly as possible as specified below.

2 [*Note:* Obtaining a value in this way can be a useful step in the process of transforming a value generated by a uniform random number generator into a value that can be delivered by a random number distribution. — *end note*]

```
template<~~class~~FloatingPointLike RealType, size_t bits, ~~class~~ UniformRandomNumberGenerator URNG>
  requires FloatingPointType<RealType>
  RealType generate_canonical(~~UniformRandomNumberGenerator~~URNG& g);
```

3 *Complexity:* Exactly $k = \max(1, \lceil b/\log_2 R \rceil)$ invocations of g, where b[4)] is the lesser of `numeric_limits<RealType>::digits` and `bits`, and R is the value of `g.max()` $-$ `g.min()` $+1$.

4 *Effects:* Invokes `g()` k times to obtain values $g_0, \ldots, g_{k-1}$, respectively. Calculates a quantity

$$S = \sum_{i=0}^{k-1} (g_i - \texttt{g.min()}) \cdot R^i$$

using arithmetic of type `RealType`.

5 *Returns:* S/R^k.

6 *Throws:* What and when g throws.

26.4.9 Random number distribution class templates [rand.dist]

1 ~~The classes and class templates specified in this section 26.4.9 satisfy all the requirements of random number distribution [rand.concept.dist]~~For every class `D` specified in this section 26.4.9 [rand.dist] or instantiated from a template specified in this section, a concept map `RandomNumberDistribution<D>` shall be defined in namespace `std` so as to provide mappings from free functions to the corresponding member functions. Descriptions are provided here only for distribution operations ~~on the distributions~~ that are not described in 26.4.3.4 [rand.concept.dist] or for operations where there is additional semantic information. Declarations for copy constructors, for copy assignment operators, and for equality and inequality operators are not shown in the synopses.

2 The algorithms for producing each of the specified distributions are implementation-defined.

3 The value of each probability density function $p(z)$ and of each discrete probability function $P(z_i)$ specified in this section is 0 everywhere outside its stated domain.

26.4.9.1 Uniform distributions [rand.dist.uni]

26.4.9.1.1 Class template `uniform_int_distribution` [rand.dist.uni.int]

1 A `uniform_int_distribution` random number distribution produces random integers i, $a \leq i \leq b$, distributed according to the constant discrete probability function

$$P(i\,|\,a,b) = 1/(b-a+1)\,.$$

```
template<~~class~~IntegralLike IntType = int>
  requires IntegralType<IntType>
  class uniform_int_distribution
{
public:
  // types
  typedef IntType result_type;
```

4) b is introduced to avoid any attempt to produce more bits of randomness than can be held in `RealType`.

```
    typedef unspecified param_type;

    // constructors and reset functions
    explicit uniform_int_distribution(IntType a = 0, IntType b = numeric_limits<IntType>::max());
    explicit uniform_int_distribution(const param_type& parm);
    void reset();

    // generating functions
    template<~~class~~ UniformRandomNumberGenerator URNG>
      result_type operator()(~~UniformRandomNumberGenerator~~URNG& g);
    template<~~class~~ UniformRandomNumberGenerator URNG>
      result_type operator()(~~UniformRandomNumberGenerator~~URNG& g, const param_type& parm);

    // property functions
    result_type a() const;
    result_type b() const;
    param_type param() const;
    void param(const param_type& parm);
    result_type min() const;
    result_type max() const;
  };
```

```
explicit uniform_int_distribution(IntType a = 0, IntType b = numeric_limits<IntType>::max());
```

2 *Requires:* $a \leq b$.

3 *Effects:* Constructs a `uniform_int_distribution` object; `a` and `b` correspond to the respective parameters of the distribution.

```
result_type a() const;
```

4 *Returns:* The value of the `a` parameter with which the object was constructed.

```
result_type b() const;
```

5 *Returns:* The value of the `b` parameter with which the object was constructed.

26.4.9.1.2 Class template `uniform_real_distribution` [rand.dist.uni.real]

1 A `uniform_real_distribution` random number distribution produces random numbers x, $a \leq x < b$, distributed according to the constant probability density function

$$p(x\,|\,a,b) = 1/(b-a)\ .$$

```
  template<~~class~~FloatingPointLike RealType = double>
    requires FloatingPointType<RealType>
    class uniform_real_distribution
  {
  public:
    // types
```

```
    typedef RealType result_type;
    typedef unspecified param_type;

    // constructors and reset functions
    explicit uniform_real_distribution(RealType a = 0.0, RealType b = 1.0);
    explicit uniform_real_distribution(const param_type& parm);
    void reset();

    // generating functions
    template<~~class~~ UniformRandomNumberGenerator URNG>
      result_type operator()(~~UniformRandomNumberGenerator~~URNG& g);
    template<~~class~~ UniformRandomNumberGenerator URNG>
      result_type operator()(~~UniformRandomNumberGenerator~~URNG& g, const param_type& parm);

    // property functions
    result_type a() const;
    result_type b() const;
    param_type param() const;
    void param(const param_type& parm);
    result_type min() const;
    result_type max() const;
  };
```

```
explicit uniform_real_distribution(RealType a = 0.0, RealType b = 1.0);
```

2 *Requires:* $\texttt{a} \le \texttt{b}$ and $\texttt{b} - \texttt{a} \le$ `numeric_limits<RealType>::max()`.

3 *Effects:* Constructs a `uniform_real_distribution` object; `a` and `b` correspond to the respective parameters of the distribution.

```
result_type a() const;
```

4 *Returns:* The value of the `a` parameter with which the object was constructed.

```
result_type b() const;
```

5 *Returns:* The value of the `b` parameter with which the object was constructed.

26.4.9.2 Bernoulli distributions [rand.dist.bern]

26.4.9.2.1 Class `bernoulli_distribution` [rand.dist.bern.bernoulli]

1 A `bernoulli_distribution` random number distribution produces `bool` values b distributed according to the discrete probability function

$$P(b\,|\,p) = \begin{cases} p & \text{if} \quad b = \texttt{true} \\ 1-p & \text{if} \quad b = \texttt{false} \end{cases} \quad .$$

```
  class bernoulli_distribution
  {
  public:
```

```
    // types
    typedef bool result_type;
    typedef unspecified param_type;

    // constructors and reset functions
    explicit bernoulli_distribution(double p = 0.5);
    explicit bernoulli_distribution(const param_type& parm);
    void reset();

    // generating functions
    template<class UniformRandomNumberGenerator URNG>
      result_type operator()(UniformRandomNumberGeneratorURNG& g);
    template<class UniformRandomNumberGenerator URNG>
      result_type operator()(UniformRandomNumberGeneratorURNG& g, const param_type& parm);

    // property functions
    double p() const;
    param_type param() const;
    void param(const param_type& parm);
    result_type min() const;
    result_type max() const;
  };
```

```
explicit bernoulli_distribution(double p = 0.5);
```

2 *Requires:* $0 \leq \mathtt{p} \leq 1$.

3 *Effects:* Constructs a `bernoulli_distribution` object; `p` corresponds to the parameter of the distribution.

```
double p() const;
```

4 *Returns:* The value of the `p` parameter with which the object was constructed.

26.4.9.2.2 Class template `binomial_distribution` [rand.dist.bern.bin]

1 A `binomial_distribution` random number distribution produces integer values $i \geq 0$ distributed according to the discrete probability function

$$P(i\,|\,t,p) = \binom{t}{i} \cdot p^i \cdot (1-p)^{t-i} .$$

```
  template<classIntegralLike IntType = int>
    requires IntegralType<IntType>
    class binomial_distribution
  {
  public:
    // types
    typedef IntType result_type;
    typedef unspecified param_type;

    // constructors and reset functions
```

```
    explicit binomial_distribution(IntType t = 1, double p = 0.5);
    explicit binomial_distribution(const param_type& parm);
    void reset();

    // generating functions
    template<~~class~~ UniformRandomNumberGenerator URNG>
      result_type operator()(~~UniformRandomNumberGenerator~~URNG& g);
    template<~~class~~ UniformRandomNumberGenerator URNG>
      result_type operator()(~~UniformRandomNumberGenerator~~URNG& g, const param_type& parm);

    // property functions
    IntType t() const;
    double p() const;
    param_type param() const;
    void param(const param_type& parm);
    result_type min() const;
    result_type max() const;
  };
```

```
explicit binomial_distribution(IntType t = 1, double p = 0.5);
```

2 *Requires:* $0 \leq \mathtt{p} \leq 1$ and $0 \leq \mathtt{t}$.

3 *Effects:* Constructs a `binomial_distribution` object; `t` and `p` correspond to the respective parameters of the distribution.

```
IntType t() const;
```

4 *Returns:* The value of the `t` parameter with which the object was constructed.

```
double p() const;
```

5 *Returns:* The value of the `p` parameter with which the object was constructed.

26.4.9.2.3 Class template `geometric_distribution` [rand.dist.bern.geo]

1 A `geometric_distribution` random number distribution produces integer values $i \geq 0$ distributed according to the discrete probability function

$$P(i \,|\, p) = p \cdot (1-p)^i \; .$$

```
  template<~~class~~IntegralLike IntType = int>
    requires IntegralType<IntType>
    class geometric_distribution
  {
  public:
    // types
    typedef IntType result_type;
    typedef unspecified param_type;

    // constructors and reset functions
```

```
explicit geometric_distribution(double p = 0.5);
explicit geometric_distribution(const param_type& parm);
void reset();

// generating functions
template<~~class~~ UniformRandomNumberGenerator URNG>
  result_type operator()(~~UniformRandomNumberGenerator~~URNG& g);
template<~~class~~ UniformRandomNumberGenerator URNG>
  result_type operator()(~~UniformRandomNumberGenerator~~URNG& g, const param_type& parm);

// property functions
double p() const;
param_type param() const;
void param(const param_type& parm);
result_type min() const;
result_type max() const;
};
```

```
explicit geometric_distribution(double p = 0.5);
```

2 *Requires:* $0 < \mathtt{p} < 1$.

3 *Effects:* Constructs a `geometric_distribution` object; `p` corresponds to the parameter of the distribution.

```
double p() const;
```

4 *Returns:* The value of the `p` parameter with which the object was constructed.

26.4.9.2.4 Class template `negative_binomial_distribution` [rand.dist.bern.negbin]

1 A `negative_binomial_distribution` random number distribution produces random integers $i \geq 0$ distributed according to the discrete probability function

$$P(i\,|\,k,p) = \binom{k+i-1}{i} \cdot p^k \cdot (1-p)^i \,.$$

```
template<~~class~~IntegralLike IntType = int>
  requires IntegralType<IntType>
  class negative_binomial_distribution
{
public:
  // types
  typedef IntType  result_type;
  typedef unspecified param_type;

  // constructor and reset functions
  explicit negative_binomial_distribution(IntType k = 1, double p = 0.5);
  explicit negative_binomial_distribution(const param_type& parm);
  void reset();
```

```
    // generating functions
    template<~~class~~ UniformRandomNumberGenerator URNG>
      result_type operator()(~~UniformRandomNumberGenerator~~URNG& g);
    template<~~class~~ UniformRandomNumberGenerator URNG>
      result_type operator()(~~UniformRandomNumberGenerator~~URNG& g, const param_type& parm);

    // property functions
    IntType k() const;
    double p() const;
    param_type param() const;
    void param(const param_type& parm);
    result_type min() const;
    result_type max() const;
  };
```

```
explicit negative_binomial_distribution(IntType k = 1, double p = 0.5);
```

2 *Requires:* $0 < \mathtt{p} \leq 1$ and $0 < \mathtt{k}$.

3 *Effects:* Constructs a `negative_binomial_distribution` object; `k` and `p` correspond to the respective parameters of the distribution.

```
IntType k() const;
```

4 *Returns:* The value of the `k` parameter with which the object was constructed.

```
double p() const;
```

5 *Returns:* The value of the `p` parameter with which the object was constructed.

26.4.9.3 Poisson distributions [rand.dist.pois]

26.4.9.3.1 Class template `poisson_distribution` [rand.dist.pois.poisson]

1 A `poisson_distribution` random number distribution produces integer values $i \geq 0$ distributed according to the discrete probability function

$$P(i\,|\,\mu) = \frac{e^{-\mu}\mu^i}{i\,!} .$$

The distribution parameter μ is also known as this distribution's *mean*.

```
  template<~~class~~IntegralLike IntType = int>
    requires IntegralType<IntType>
    class poisson_distribution
  {
  public:
    // types
    typedef IntType result_type;
    typedef unspecified param_type;
```

```
    // constructors and reset functions
    explicit poisson_distribution(double mean = 1.0);
    explicit poisson_distribution(const param_type& parm);
    void reset();

    // generating functions
    template<class UniformRandomNumberGenerator URNG>
      result_type operator()(UniformRandomNumberGeneratorURNG& g);
    template<class UniformRandomNumberGenerator URNG>
      result_type operator()(UniformRandomNumberGeneratorURNG& g, const param_type& parm);

    // property functions
    double mean() const;
    param_type param() const;
    void param(const param_type& parm);
    result_type min() const;
    result_type max() const;
  };
```

```
explicit poisson_distribution(double mean = 1.0);
```

2 *Requires:* $0 <$ `mean`.

3 *Effects:* Constructs a `poisson_distribution` object; `mean` corresponds to the parameter of the distribution.

```
double mean() const;
```

4 *Returns:* The value of the mean parameter with which the object was constructed.

26.4.9.3.2 Class template `exponential_distribution` [rand.dist.pois.exp]

1 An `exponential_distribution` random number distribution produces random numbers $x > 0$ distributed according to the probability density function

$$p(x\,|\,\lambda) = \lambda e^{-\lambda x} .$$

```
  template<classFloatingPointLike RealType = double>
    requires FloatingPointType<RealType>
    class exponential_distribution
  {
  public:
    // types
    typedef RealType result_type;
    typedef unspecified param_type;

    // constructors and reset functions
    explicit exponential_distribution(RealType lambda = 1.0);
    explicit exponential_distribution(const param_type& parm);
    void reset();

    // generating functions
```

```
  template<class UniformRandomNumberGenerator URNG>
    result_type operator()(UniformRandomNumberGeneratorURNG& g);
  template<class UniformRandomNumberGenerator URNG>
    result_type operator()(UniformRandomNumberGeneratorURNG& g, const param_type& parm);

  // property functions
  RealType lambda() const;
  param_type param() const;
  void param(const param_type& parm);
  result_type min() const;
  result_type max() const;
};
```

```
explicit exponential_distribution(RealType lambda = 1.0);
```

2 *Requires:* $0 <$ `lambda`.

3 *Effects:* Constructs a `exponential_distribution` object; `lambda` corresponds to the parameter of the distribution.

```
RealType lambda() const;
```

4 *Returns:* The value of the `lambda` parameter with which the object was constructed.

26.4.9.3.3 Class template `gamma_distribution` [rand.dist.pois.gamma]

1 A `gamma_distribution` random number distribution produces random numbers $x > 0$ distributed according to the probability density function

$$p(x \mid \alpha, \beta) = \frac{e^{-x/\beta}}{\beta^{\alpha} \cdot \Gamma(\alpha)} \cdot x^{\alpha-1} .$$

```
template<classFloatingPointLike RealType = double>
  requires FloatingPointType<RealType>
  class gamma_distribution
{
public:
  // types
  typedef RealType result_type;
  typedef unspecified param_type;

  // constructors and reset functions
  explicit gamma_distribution(RealType alpha = 1.0, RealType beta = 1.0);
  explicit gamma_distribution(const param_type& parm);
  void reset();

  // generating functions
  template<class UniformRandomNumberGenerator URNG>
    result_type operator()(UniformRandomNumberGeneratorURNG& g);
  template<class UniformRandomNumberGenerator URNG>
```

```
    result_type operator()(UniformRandomNumberGeneratorURNG& g, const param_type& parm);

  // property functions
  RealType alpha() const;
  RealType beta() const;
  param_type param() const;
  void param(const param_type& parm);
  result_type min() const;
  result_type max() const;
};
```

```
explicit gamma_distribution(RealType alpha = 1.0, RealType beta = 1.0);
```

2 *Requires:* $0 <$ `alpha` and $0 <$ `beta`.

3 *Effects:* Constructs a `gamma_distribution` object; `alpha` and `beta` correspond to the parameters of the distribution.

```
RealType alpha() const;
```

4 *Returns:* The value of the `alpha` parameter with which the object was constructed.

```
RealType beta() const;
```

5 *Returns:* The value of the `beta` parameter with which the object was constructed.

26.4.9.3.4 Class template `weibull_distribution` [rand.dist.pois.weibull]

1 A `weibull_distribution` random number distribution produces random numbers $x \geq 0$ distributed according to the probability density function

$$p(x\,|\,a,b) = \frac{a}{b} \cdot \left(\frac{x}{b}\right)^{a-1} \cdot \exp\left(-\left(\frac{x}{b}\right)^{a}\right) .$$

```
template<classFloatingPointLike RealType = double>
  requires FloatingPointType<RealType>
  class weibull_distribution
{
public:
  // types
  typedef RealType result_type;
  typedef unspecified param_type;

  // constructor and reset functions
  explicit weibull_distribution(RealType a = 1.0, RealType b = 1.0)
  explicit weibull_distribution(const param_type& parm);
  void reset();

  // generating functions
  template<class UniformRandomNumberGenerator URNG>
    result_type operator()(UniformRandomNumberGeneratorURNG& g);
```

```
  template<~~class~~ UniformRandomNumberGenerator URNG>
    result_type operator()(~~UniformRandomNumberGenerator~~URNG& g, const param_type& parm);

  // property functions
  RealType a() const;
  RealType b() const;
  param_type param() const;
  void param(const param_type& parm);
  result_type min() const;
  result_type max() const;
};
```

`explicit weibull_distribution(RealType a = 1.0, RealType b = 1.0);`

2 *Requires:* $0 <$ a and $0 <$ b.

3 *Effects:* Constructs a `weibull_distribution` object; a and b correspond to the respective parameters of the distribution.

`RealType a() const;`

4 *Returns:* The value of the a parameter with which the object was constructed.

`RealType b() const;`

5 *Returns:* The value of the b parameter with which the object was constructed.

26.4.9.3.5 Class template `extreme_value_distribution` [rand.dist.pois.extreme]

1 An `extreme_value_distribution` random number distribution produces random numbers x distributed according to the probability density function[5)]

$$p(x\,|\,a,b) = \frac{1}{b} \cdot \exp\left(\frac{a-x}{b} - \exp\left(\frac{a-x}{b}\right)\right) .$$

```
template<~~class~~FloatingPointLike RealType = double>
  requires FloatingPointType<RealType>
  class extreme_value_distribution
{
public:
  // types
  typedef RealType result_type;
  typedef unspecified param_type;

  // constructor and reset functions
  explicit extreme_value_distribution(RealType a = 0.0, RealType b = 1.0);
  explicit extreme_value_distribution(const param_type& parm);
```

5) The distribution corresponding to this probability density function is also known (with a possible change of variable) as the Gumbel Type I, the log-Weibull, or the Fisher-Tippett Type I distribution.

```
    void reset();

    // generating functions
    template<class UniformRandomNumberGenerator URNG>
      result_type operator()(UniformRandomNumberGeneratorURNG& g);
    template<class UniformRandomNumberGenerator URNG>
      result_type operator()(UniformRandomNumberGeneratorURNG& g, const param_type& parm);

    // property functions
    RealType a() const;
    RealType b() const;
    param_type param() const;
    void param(const param_type& parm);
    result_type min() const;
    result_type max() const;
  };
```

```
explicit extreme_value_distribution(RealType a = 0.0, RealType b = 1.0);
```

2 *Requires:* $0 < \texttt{b}$.

3 *Effects:* Constructs an `extreme_value_distribution` object; `a` and `b` correspond to the respective parameters of the distribution.

```
RealType a() const;
```

4 *Returns:* The value of the `a` parameter with which the object was constructed.

```
RealType b() const;
```

5 *Returns:* The value of the `b` parameter with which the object was constructed.

26.4.9.4 Normal distributions [rand.dist.norm]

26.4.9.4.1 Class template `normal_distribution` [rand.dist.norm.normal]

1 A `normal_distribution` random number distribution produces random numbers x distributed according to the probability density function

$$p(x\,|\,\mu,\sigma)p(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$$

The distribution parameters μ and σ are also known as this distribution's *mean* and *standard deviation*.

```
  template<classFloatingPointLike RealType = double>
    requires FloatingPointType<RealType>
    class normal_distribution
  {
  public:
    // types
    typedef RealType result_type;
    typedef unspecified param_type;
```

```
  // constructors and reset functions
  explicit normal_distribution(RealType mean = 0.0, RealType stddev = 1.0);
  explicit normal_distribution(const param_type& parm);
  void reset();

  // generating functions
  template<~~class~~ UniformRandomNumberGenerator URNG>
    result_type operator()(~~UniformRandomNumberGenerator~~URNG& g);
  template<~~class~~ UniformRandomNumberGenerator URNG>
    result_type operator()(~~UniformRandomNumberGenerator~~URNG& g, const param_type& parm);

  // property functions
  RealType mean() const;
  RealType stddev() const;
  param_type param() const;
  void param(const param_type& parm);
  result_type min() const;
  result_type max() const;
};
```

`explicit normal_distribution(RealType mean = 0.0, RealType stddev = 1.0);`

2 *Requires:* $0 <$ `stddev`.

3 *Effects:* Constructs a `normal_distribution` object; `mean` and `stddev` correspond to the respective parameters of the distribution.

`RealType mean() const;`

4 *Returns:* The value of the `mean` parameter with which the object was constructed.

`RealType stddev() const;`

5 *Returns:* The value of the `stddev` parameter with which the object was constructed.

26.4.9.4.2 Class template `lognormal_distribution` [rand.dist.norm.lognormal]

1 A `lognormal_distribution` random number distribution produces random numbers $x > 0$ distributed according to the probability density function

$$p(x\,|\,m,s) = \frac{1}{sx\sqrt{2\pi}} \cdot \exp\left(-\frac{(\ln x - m)^2}{2s^2}\right).$$

```
template<~~class~~FloatingPointLike RealType = double>
  requires FloatingPointType<RealType>
  class lognormal_distribution
{
public:
  // types
  typedef RealType result_type;
```

```
    typedef unspecified param_type;

    // constructor and reset functions
    explicit lognormal_distribution(RealType m = 0.0, RealType s = 1.0);
    explicit lognormal_distribution(const param_type& parm);
    void reset();

    // generating functions
    template<~~class~~ UniformRandomNumberGenerator URNG>
      result_type operator()(~~UniformRandomNumberGenerator~~URNG& g);
    template<~~class~~ UniformRandomNumberGenerator URNG>
      result_type operator()(~~UniformRandomNumberGenerator~~URNG& g, const param_type& parm);

    // property functions
    RealType m() const;
    RealType s() const;
    param_type param() const;
    void param(const param_type& parm);
    result_type min() const;
    result_type max() const;
  };
```

```
explicit lognormal_distribution(RealType m = 0.0, RealType s = 1.0);
```

2 *Requires:* $0 < \mathtt{s}$.

3 *Effects:* Constructs a `lognormal_distribution` object; `m` and `s` correspond to the respective parameters of the distribution.

```
RealType m() const;
```

4 *Returns:* The value of the `m` parameter with which the object was constructed.

```
RealType s() const;
```

5 *Returns:* The value of the `s` parameter with which the object was constructed.

26.4.9.4.3 Class template `chi_squared_distribution` [rand.dist.norm.chisq]

1 A `chi_squared_distribution` random number distribution produces random numbers $x > 0$ distributed according to the probability density function

$$p(x\,|\,n) = \frac{x^{(n/2)-1} \cdot e^{-x/2}}{\Gamma(n/2) \cdot 2^{n/2}} \ ,$$

where n is a positive integer.

```
  template<~~class~~FloatingPointLike RealType = double>
    requires FloatingPointType<RealType>
    class chi_squared_distribution
  {
  public:
```

```
    // types
    typedef RealType result_type;
    typedef unspecified param_type;

    // constructor and reset functions
    explicit chi_squared_distribution(int n = 1);
    explicit chi_squared_distribution(const param_type& parm);
    void reset();

    // generating functions
    template<~~class~~ UniformRandomNumberGenerator URNG>
      result_type operator()(~~UniformRandomNumberGenerator~~URNG& g);
    template<~~class~~ UniformRandomNumberGenerator URNG>
      result_type operator()(~~UniformRandomNumberGenerator~~URNG& g, const param_type& parm);

    // property functions
    int n() const;
    param_type param() const;
    void param(const param_type& parm);
    result_type min() const;
    result_type max() const;
  };
```

`explicit chi_squared_distribution(int n = 1);`

2 *Requires:* $0 <$ `n`.

3 *Effects:* Constructs a `chi_squared_distribution` object; `n` corresponds to the parameter of the distribution.

`int n() const;`

4 *Returns:* The value of the `n` parameter with which the object was constructed.

26.4.9.4.4 Class template `cauchy_distribution` [rand.dist.norm.cauchy]

1 A `cauchy_distribution` random number distribution produces random numbers x distributed according to the probability density function

$$p(x\,|\,a,b) = \left(\pi b \left(1 + \left(\frac{x-a}{b}\right)^2\right)\right)^{-1} .$$

```
  template<~~class~~FloatingPointLike RealType = double>
    requires FloatingPointType<RealType>
    class cauchy_distribution
  {
  public:
    // types
    typedef RealType result_type;
    typedef unspecified param_type;
```

```
    // constructor and reset functions
    explicit cauchy_distribution(RealType a = 0.0, RealType b = 1.0);
    explicit cauchy_distribution(const param_type& parm);
    void reset();

    // generating functions
    template<class UniformRandomNumberGenerator URNG>
      result_type operator()(UniformRandomNumberGeneratorURNG& g);
    template<class UniformRandomNumberGenerator URNG>
      result_type operator()(UniformRandomNumberGeneratorURNG& g, const param_type& parm);

    // property functions
    RealType a() const;
    RealType b() const;
    param_type param() const;
    void param(const param_type& parm);
    result_type min() const;
    result_type max() const;
  };
```

```
explicit cauchy_distribution(RealType a = 0.0, RealType b = 1.0);
```

2 *Requires:* $0 < \texttt{b}$.

3 *Effects:* Constructs a `cauchy_distribution` object; `a` and `b` correspond to the respective parameters of the distribution.

```
RealType a() const;
```

4 *Returns:* The value of the `a` parameter with which the object was constructed.

```
RealType b() const;
```

5 *Returns:* The value of the `b` parameter with which the object was constructed.

26.4.9.4.5 Class template `fisher_f_distribution` [rand.dist.norm.f]

1 A `fisher_f_distribution` random number distribution produces random numbers $x \geq 0$ distributed according to the probability density function

$$p(x\,|\,m,n) = \frac{\Gamma\big((m+n)/2\big)}{\Gamma(m/2)\,\Gamma(n/2)} \cdot \left(\frac{m}{n}\right)^{m/2} \cdot x^{(m/2)-1} \cdot \left(1 + \frac{mx}{n}\right)^{-(m+n)/2},$$

where m and n are positive integers.

```
  template<classFloatingPointLike RealType = double>
    requires FloatingPointType<RealType>
    class fisher_f_distribution
  {
  public:
    // types
```

```
    typedef RealType result_type;
    typedef unspecified param_type;

    // constructor and reset functions
    explicit fisher_f_distribution(int m = 1, int n = 1);
    explicit fisher_f_distribution(const param_type& parm);
    void reset();

    // generating functions
    template<class UniformRandomNumberGenerator URNG>
      result_type operator()(UniformRandomNumberGeneratorURNG& g);
    template<class UniformRandomNumberGenerator URNG>
      result_type operator()(UniformRandomNumberGeneratorURNG& g, const param_type& parm);

    // property functions
    int m() const;
    int n() const;
    param_type param() const;
    void param(const param_type& parm);
    result_type min() const;
    result_type max() const;
  };
```

```
explicit fisher_f_distribution(int m = 1, int n = 1);
```

2 *Requires:* $0 < \mathtt{m}$ and $0 < \mathtt{n}$.

3 *Effects:* Constructs a `fisher_f_distribution` object; `m` and `n` correspond to the respective parameters of the distribution.

```
int m() const;
```

4 *Returns:* The value of the `m` parameter with which the object was constructed.

```
int n() const;
```

5 *Returns:* The value of the `n` parameter with which the object was constructed.

26.4.9.4.6 Class template `student_t_distribution` [rand.dist.norm.t]

1 A `student_t_distribution` random number distribution produces random numbers x distributed according to the probability density function

$$p(x\,|\,n) = \frac{1}{\sqrt{n\pi}} \cdot \frac{\Gamma\big((n+1)/2\big)}{\Gamma(n/2)} \cdot \left(1+\frac{x^2}{n}\right)^{-(n+1)/2} ,$$

where n is a positive integer.

```
  template<classFloatingPointLike RealType = double>
    requires FloatingPointType<RealType>
    class student_t_distribution
  {
```

```
  public:
    // types
    typedef RealType result_type;
    typedef unspecified param_type;

    // constructor and reset functions
    explicit student_t_distribution(int n = 1);
    explicit student_t_distribution(const param_type& parm);
    void reset();

    // generating functions
    template<~~class~~ UniformRandomNumberGenerator URNG>
      result_type operator()(~~UniformRandomNumberGenerator~~URNG& g);
    template<~~class~~ UniformRandomNumberGenerator URNG>
      result_type operator()(~~UniformRandomNumberGenerator~~URNG& g, const param_type& parm);

    // property functions
    int n() const;
    param_type param() const;
    void param(const param_type& parm);
    result_type min() const;
    result_type max() const;
  };
```

```
explicit student_t_distribution(int n = 1);
```

2 *Requires:* $0 <$ `n`.

3 *Effects:* Constructs a `student_t_distribution` object; `n` and `n` correspond to the respective parameters of the distribution.

```
int n() const;
```

4 *Returns:* The value of the `n` parameter with which the object was constructed.

26.4.9.5 Sampling distributions [rand.dist.samp]

26.4.9.5.1 Class template `discrete_distribution` [rand.dist.samp.discrete]

1 A `discrete_distribution` random number distribution produces random integers i, $0 \leq i < n$, distributed according to the discrete probability function

$$P(i \,|\, p_0, \ldots, p_{n-1}) = p_i \,.$$

```
  template<~~class~~IntegralLike IntType = int>
    requires IntegralType<IntType>
    class discrete_distribution
  {
  public:
    // types
    typedef IntType result_type;
```

```
    typedef unspecified param_type;

    // constructor and reset functions
    discrete_distribution();
    template<~~class~~ InputIterator Iter>
      requires Convertible<Iter::value_type, double>
      discrete_distribution(~~InputIterator~~Iter firstW, ~~InputIterator~~Iter lastW);
    explicit discrete_distribution(const param_type& parm);
    void reset();

    // generating functions
    template<~~class~~ UniformRandomNumberGenerator URNG>
      result_type operator()(~~UniformRandomNumberGenerator~~URNG& g);
    template<~~class~~ UniformRandomNumberGenerator URNG>
      result_type operator()(~~UniformRandomNumberGenerator~~URNG& g, const param_type& parm);

    // property functions
    vector<double> probabilities() const;
    param_type param() const;
    void param(const param_type& parm);
    result_type min() const;
    result_type max() const;
  };
```

```
discrete_distribution();
```

2 *Effects:* Constructs a `discrete_distribution` object with $n = 1$ and $p_0 = 1$. [*Note:* Such an object will always deliver the value 0. — *end note*]

```
template<~~class~~ InputIterator Iter>r
  requires Convertible<Iter::value_type, double>
  discrete_distribution(~~InputIterator~~Iter firstW, ~~InputIterator~~Iter lastW);
```

3 *Requires:*

a) ~~`InputIterator` shall satisfy the requirements of an input iterator [input.iterator].~~

b) If `firstW == lastW`, let the sequence w have length $n = 1$ and consist of the single value $w_0 = 1$. Otherwise, `[firstW,lastW)` shall form a sequence w of length $n > 0$ ~~and `*firstW` shall yield a value w_0 convertible to `double`~~. [*Note:* The values w_k are commonly known as the *weights*. — *end note*]

c) The following relations shall hold: $w_k \geq 0$ for $k = 0, \ldots, n-1$, and $0 < S = w_0 + \cdots + w_{n-1}$.

4 *Effects:* Constructs a `discrete_distribution` object with probabilities

$$p_k = \frac{w_k}{S} \text{ for } k = 0, \ldots, n-1.$$

```
vector<double> probabilities() const;
```

5 *Returns:* A `vector<double>` whose `size` member returns n and whose `operator[]` member returns p_k when invoked with argument k for $k = 0, \ldots, n-1$.

26.4.9.5.2 Class template `piecewise_constant_distribution` [rand.dist.samp.pconst]

1 A `piecewise_constant_distribution` random number distribution produces random numbers x, $b_0 \le x < b_n$, uniformly distributed over each subinterval $[b_i, b_{i+1})$ according to the probability density function

$$p(x \mid b_0, \ldots, b_n,\ \rho_0, \ldots, \rho_{n-1}) = \rho_i \text{ , for } b_i \le x < b_{i+1} \text{ .}$$

The $n+1$ distribution parameters b_i are also known as this distribution's *interval boundaries*.

```
template<classFloatingPointLike RealType = double>
  requires FloatingPointType<RealType>
  class piecewise_constant_distribution
{
public:
  // types
  typedef RealType result_type;
  typedef unspecified param_type;

  // constructor and reset functions
  piecewise_constant_distribution();
  template<class InputIteratorBInputIterator IterB, class InputIteratorWInputIterator IterW>
    requires Convertible<IterB::value_type, result_type> && Convertible<IterW::value_type, double>
    piecewise_constant_distribution(InputIteratorBIterB firstB, InputIteratorBIterB lastB,
                                    InputIteratorWIterW firstW);
  explicit piecewise_constant_distribution(const param_type& parm);
  void reset();

  // generating functions
  template<class UniformRandomNumberGenerator URNG>
    result_type operator()(UniformRandomNumberGeneratorURNG& g);
  template<class UniformRandomNumberGenerator URNG>
    result_type operator()(UniformRandomNumberGeneratorURNG& g, const param_type& parm);

  // property functions
  vector<RealType> intervals() const;
  vector<double> densities() const;
  param_type param() const;
  void param(const param_type& parm);
  result_type min() const;
  result_type max() const;
};
```

```
piecewise_constant_distribution();
```

2 *Effects:* Constructs a `piecewise_constant_distribution` object with $n = 1$, $\rho_0 = 1$, $b_0 = 0$, and $b_1 = 1$.

```
template<class InputIteratorBInputIterator IterB, class InputIteratorWInputIterator IterW>
  requires Convertible<IterB::value_type, result_type> && Convertible<IterW::value_type, double>
  piecewise_constant_distribution(InputIteratorBIterB firstB, InputIteratorBIterB lastB,
                                  InputIteratorWIterW firstW);
```

3 *Requires:*

a) ~~InputIteratorB shall satisfy the requirements of an input iterator [input.iterator], as shall InputIteratorW.~~

b) If `firstB == lastB` or the sequence `w` has the length zero,

(a) let the sequence w have length $n = 1$ and consist of the single value $w_0 = 1$, and

(b) let the sequence b have length $n+1$ with $b_0 = 0$ and $b_1 = 1$.

Otherwise,

(c) `[firstB, lastB)` shall form a sequence b of length $n+1$ ~~whose leading element b_0 shall be convertible to result_type~~, and

(d) the length of the sequence w starting from `firstW` shall be at least n, ~~*firstW shall return a value w_0 that is convertible to double,~~ and any w_k for $k \geq n$ shall be ignored by the distribution.

[*Note:* The values w_k are commonly known as the *weights*. — *end note*]

c) The following relations shall hold for $k = 0, \ldots, n-1$: $b_k < b_{k+1}$ and $0 \leq w_k$. Also, $0 < S = w_0 + \cdots + w_{n-1}$.

4 *Effects:* Constructs a `piecewise_constant_distribution` object with probability densities

$$\rho_k = \frac{w_k}{S \cdot (b_{k+1} - b_k)} \quad \text{for } k = 0, \ldots, n-1.$$

```
vector<result_type> intervals() const;
```

5 *Returns:* A `vector<result_type>` whose `size` member returns $n+1$ and whose `operator[]` member returns b_k when invoked with argument k for $k = 0, \ldots, n$.

```
vector<double> densities() const;
```

6 *Returns:* A `vector<result_type>` whose `size` member returns n and whose `operator[]` member returns ρ_k when invoked with argument k for $k = 0, \ldots, n-1$.

26.4.9.5.3 Class template `general_pdf_distribution` [rand.dist.samp.genpdf]

1 A `general_pdf_distribution` random number distribution produces random numbers x, $x_{\min} \leq x < x_{\max}$, distributed according to the probability density function

$$p(x \,|\, x_{\min}, x_{\max}, \rho) = \rho(x) \text{ , for } x_{\min} \leq x < x_{\min} \text{ .}$$

```
template<classFloatingPointLike RealType = double>
  requires FloatingPointType<RealType>
  class general_pdf_distribution
{
public:
  // types
  typedef RealType result_type;
```

```
  typedef unspecified param_type;

  // constructor and reset functions
  general_pdf_distribution();
  template<classCallable<auto,result_type> Func>
    requires Convertible<Func::result_type, double>
    general_pdf_distribution(result_type xmin, result_type xmax, Func pdf);
  explicit general_pdf_distribution(const param_type& parm);
  void reset();

  // generating functions
  template<class UniformRandomNumberGenerator URNG>
    result_type operator()(UniformRandomNumberGeneratorURNG& g);
  template<class UniformRandomNumberGenerator URNG>
    result_type operator()(UniformRandomNumberGeneratorURNG& g, const param_type& parm);

  // property functions
  result_type xmin() const;
  result_type xmax() const;
  param_type param() const;
  void param(const param_type& parm);
  result_type min() const;
  result_type max() const;
};
```

```
general_pdf_distribution();
```

2 *Effects:* Constructs a `general_pdf_distribution` object with $x_{min} = 0$ and $x_{max} = 1$ such that $p(x) = 1$ for all $x_{min} \leq x < x_{max}$.

```
template<classCallable<auto,result_type> Func>
  requires Convertible<Func::result_type, double>
  general_pdf_distribution(result_type xmin, result_type xmax, Func pdf);
```

3 *Requires:*

a) ~~pdf shall be callable with one argument of type result_type, and shall return values of a type convertible to double;~~

b) $x_{\min} < x_{\max}$, and for all $x_{\min} \leq x < x_{\max}$, `pdf`$(x)$ shall return a value that is non-negative, non-NaN, and non-infinity; and

c) the following relations shall hold:

$$0 < z = \int_{x_{\min}}^{x_{\max}} f(x)\, dx < \infty\,,$$

where f is the mathematical function corresponding to the supplied `pdf`. [*Note:* This implies that the user-supplied `pdf` need not be normalized. — *end note*]

4 *Effects:* Constructs a `general_pdf_distribution` object; `xmin` and `xmax` correspond to the respective parameters of the distribution and the corresponding probability density function is given by $\rho(x) = f(x)/z$.

```
result_type xmin() const;
```

5 *Returns:* The value of the `xmin` parameter with which the object was constructed.

```
result_type xmax() const;
```

6 *Returns:* The value of the `xmax` parameter with which the object was constructed.

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